Eureka Lesson for 6th Grade Unit ONE

Models of Dividing Fractions

*These 2 lessons can be taught in 2 class periods – Or 3 with struggling learners.*

Challenges: We (middle school teachers) are not comfortable teaching operations with fractions using models, but these 2 lessons are well laid out with step-by-step instructions to make it easier. Please familiarize yourselves with the lesson before using it.

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Topic A:

Dividing Fractions by Fractions

6.NS.A.1

Focus Standard: 6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$). How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many $3/4$-cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?

Instructional Days: 8

Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models (P)

Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models (P)

Lessons 3–4: Interpreting and Computing Division of a Fraction by a Fraction—More Models (P)

Lesson 5: Creating Division Stories (P)

Lesson 6: More Division Stories (P)

Lesson 7: The Relationship Between Visual Fraction Models and Equations (S)

Lesson 8: Dividing Fractions and Mixed Numbers (P)

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. Students determine quotients through visual models, such as bar diagrams, tape diagrams, arrays, and number line diagrams. They construct division stories and solve word problems involving division of fractions (6.NS.A.1). Students understand and apply partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask themselves, “How much is one share?” and “What part of the unit is that share?” Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Later in the module, students will understand and apply the direct correlation of division of fractions to division of decimals.

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1 Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson
Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Student Outcomes

- Students use visual models, such as fraction bars, number lines, and area models, to show the quotient of whole numbers and fractions and to show the connection between them and the multiplication of fractions.
- Students divide a fraction by a whole number.

Classwork

Opening Exercise (5 minutes)

At the beginning of class, hand each student a fraction card (see page 18). Ask students to do the following Opening Exercise.

<table>
<thead>
<tr>
<th>Opening Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw a model of the fraction.</td>
</tr>
<tr>
<td>Describe what the fraction means.</td>
</tr>
</tbody>
</table>

After two minutes, have students share some of their models and descriptions. Emphasize the key point that a fraction shows division of the numerator by the denominator. In other words, a fraction shows a part being divided by a whole. Also, remind students that fractions are numbers; therefore, they can be added, subtracted, multiplied, or divided.

To conclude the Opening Exercise, students can share where their fractions would be located on a number line. A number line can be drawn on a chalkboard or projected onto a board. Then, students can describe how the fractions on the cards would be placed in order on the number line.

Example 1 (7 minutes)

This lesson will focus on fractions divided by whole numbers. Students learned how to divide unit fractions by whole numbers in Grade 5. Teachers can become familiar with what was taught on this topic by reviewing the materials used in the Grade 5, Module 4 lessons and assessments.

Scaffolding:
Each class should have a set of fraction tiles. Students who are struggling may benefit from using the fraction tiles to see the division until they are better at drawing the models.
Example 1
Maria has $\frac{3}{4}$ lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? What is this question asking us to do?

*We are being asked to divide the trail mix into six equal portions. So, we need to divide three-fourths by six.*

How can this question be modeled?

- Let’s take a look at how to solve this using a number line and a fraction bar.

We will start by creating a number line broken into fourths and a fraction bar broken into fourths.

![Number Line](image1)

![Fraction Bar](image2)

- We are going to give equal amounts of trail mix to each person. How can we show this in the model?
  - *We will divide the shaded portion so that it includes six equal-sized pieces.*

- How will we show this on the number line?
  - *There are three equal sections on the number line that also need to be divided into six equal shares.*

Next, we need to determine the unit. What did we do to each of the three sections in the fraction bar?

- *We divided them into two pieces.*

What should we do to the remaining piece of the fraction bar?

- *Divide it into two pieces.*

How many pieces are there total?

- *8 pieces*
What does each piece or section represent?

- \( \frac{1}{8} \)

Therefore, \( \frac{3}{4} \div 6 = \frac{1}{8} \). This visual model also shows that \( \frac{1}{6} \) of \( \frac{3}{4} \) is \( \frac{1}{8} \).

This is an example of partitive division. You can tell because we were given the original amount of trail mix and how many “parts” of trail mix to make. We needed to determine the size of each part, where the size of each part is less than the original amount.

Example 2 (7 minutes)

Let’s look at a slightly different example. Imagine that you have \( \frac{2}{5} \) of a cup of frosting to share equally among three desserts. How would we write this as a division question?

\( \frac{2}{5} \div 3 \)

We can start by drawing a model of two-fifths.

How can we show that we are dividing two-fifths into three equal parts?
Lesson 1:

What does this part represent?

*From the visual model, we can determine that $\frac{2}{5} ÷ 3 = \frac{2}{15}.*

Exercises 1–5 (16 minutes)

Students will work in pairs to solve the following questions.

**Exercises 1–5**

For each question below, rewrite the problem as a multiplication question. Then, model the answer.

1. \[ \frac{1}{2} ÷ 6 = \frac{1}{12} \]

   \[
   \begin{array}{c}
   \text{1 of 2 is } \frac{1}{12}.
   \end{array}
   \]

   *I need to divide \( \frac{1}{2} \) into 6 equal sections. Or, I need to rewrite the problem as \( \frac{1}{6} \text{ of } \frac{1}{2} \).*

2. \[ \frac{1}{3} ÷ 3 = \frac{1}{9} \]

   \[
   \begin{array}{c}
   \text{1 of 3 is } \frac{1}{9}.
   \end{array}
   \]

   *I need to divide \( \frac{1}{3} \) into 3 equal sections. Or, I need to rewrite the problem as \( \frac{1}{3} \text{ of } \frac{1}{3} \).*

3. \[ \frac{1}{5} ÷ 4 = \frac{1}{20} \]

   \[
   \begin{array}{c}
   \text{1 of 5 is } \frac{1}{20} \text{ or } \frac{1}{4} \text{ of } \frac{1}{5}.
   \end{array}
   \]

   *I need to divide \( \frac{1}{5} \) into 4 equal sections. Or, I need to rewrite the problem as \( \frac{1}{4} \text{ of } \frac{1}{5} \).*
4. \( \frac{3}{5} \div 4 = \frac{3}{20} \)

I need to divide \( \frac{3}{5} \) into 4 equal sections. Or, I need to rewrite the problem as \( \frac{1}{4} \) of \( \frac{3}{5} \).

5. \( \frac{2}{3} \div 4 = \frac{2}{12} \) or \( \frac{1}{6} \)

I need to divide \( \frac{2}{3} \) into 4 equal sections. Or, I need to rewrite the problem as \( \frac{1}{4} \) of \( \frac{2}{3} \).

Closing (5 minutes)

- When a fraction is divided by a whole number, how does the answer compare with the dividend (the original fraction)?
  - Students should notice that the quotient is smaller than the original fraction.

Exit Ticket (5 minutes)
Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Exit Ticket

Find the quotient using a model.

1. \( \frac{2}{3} \div 3 \)

2. \( \frac{5}{6} \div 2 \)
Exit Ticket Sample Solutions

Solve each division problem using a model.

1. \( \frac{2}{3} \div 3 = \frac{2}{9} \)

2. \( \frac{5}{6} \div 2 = \frac{5}{12} \)

Problem Set Sample Solutions

Rewrite each problem as a multiplication question. Model your answer.

1. \( \frac{2}{5} \div 5 \)

I need to find \( \frac{1}{5} \) of \( \frac{2}{5} \). I would get \( \frac{2}{25} \).
2. \( \frac{3}{4} \div 2 \)

I need to find \( \frac{1}{2} \) of \( \frac{3}{4} \). I would get \( \frac{3}{8} \).
Fraction cards to use at the beginning of class:

<table>
<thead>
<tr>
<th>1</th>
<th>3</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models
Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Classwork

Opening Exercise

Draw a model of the fraction.

Describe what the fraction means.

Example 1

Maria has \( \frac{3}{4} \) lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? What is this question asking us to do?

How can this question be modeled?
Example 2

Let’s look at a slightly different example. Imagine that you have \( \frac{2}{5} \) of a cup of frosting to share equally among three desserts. How would we write this as a division question?

We can start by drawing a model of two-fifths.

\[
\begin{array}{cccc}
& & & \\
\hline
& & & \\
\hline
& & & \\
\hline
& & & \\
\hline
& & & \\
\hline
\end{array}
\]

How can we show that we are dividing two-fifths into three equal parts?

What does this part represent?

Exercises 1–5

For each question below, rewrite the problem as a multiplication question. Then, model the answer.

1. \( \frac{1}{2} \div 6 = \)
2. \( \frac{1}{3} \div 3 = \)

3. \( \frac{1}{5} \div 4 = \)

4. \( \frac{3}{5} \div 4 = \)

5. \( \frac{2}{3} \div 4 = \)
Problem Set

Rewrite each problem as a multiplication question. Model your answer.

1. \( \frac{2}{5} \div 5 \)

2. \( \frac{3}{4} \div 2 \)
Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Student Outcomes

- Students use fraction bars, number lines, and area models to show the quotient of whole numbers and fractions and to show the connection between those models and the multiplication of fractions.
- Students understand the difference between a whole number being divided by a fraction and a fraction being divided by a whole number.

Classwork

Example 1 (15 minutes)

At the beginning of class, break students into groups. Each group will need to answer the question they have been assigned and draw a model to represent their answer. Multiple groups could have the same question.

Group 1: How many half-miles are in 12 miles? $12 \div \frac{1}{2} = 24$

Group 2: How many quarter hours are in 5 hours? $5 \div \frac{1}{4} = 20$

Group 3: How many one-third cups are in 9 cups? $9 \div \frac{1}{3} = 27$

Group 4: How many one-eighth pizzas are in 4 pizzas? $4 \div \frac{1}{8} = 32$

Group 5: How many one-fifths are in 7 wholes? $7 \div \frac{1}{5} = 35$

Models will vary, but could include fraction bars, number lines, or area models (arrays).

Students will draw models on blank paper, construction paper, or chart paper. Hang up only student models, and have students travel around the room answering the following:

1. Write the division question that was answered with each model.
2. What multiplication question could this model also answer?
3. Rewrite the question given to each group as a multiplication question.

Students will be given a table to fill in as they visit each model.

When discussing the opening of this example, ask students how these questions are different from the questions solved in Lesson 1. Students should notice that these questions are dividing whole numbers by fractions, while the questions in Lesson 1 were dividing fractions by whole numbers.

Discuss how the division problem is related to the multiplication problem. Students should recognize that when 12 is divided into halves, it is the same as doubling 12.
Example 1

Question # _______

Write it as a division question. ____________________________________________

Write it as a multiplication question. _________________________________________

Make a rough draft of a model to represent the question:

As you travel to each model, be sure to answer the following questions:

<table>
<thead>
<tr>
<th>Original Questions</th>
<th>Write the division question that was answered in each model.</th>
<th>What multiplication question could the model also answer?</th>
<th>Write the question given to each group as a multiplication question.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How many ( \frac{1}{2} ) miles are in 12 miles?</td>
<td>( 12 \div \frac{1}{2} )</td>
<td>( 12 \times 2 = ? )</td>
<td>Answers will vary.</td>
</tr>
<tr>
<td>2. How many quarter hours are in 5 hours?</td>
<td>( 5 \div \frac{1}{4} )</td>
<td>( 5 \times 4 = ? )</td>
<td></td>
</tr>
<tr>
<td>3. How many ( \frac{1}{3} ) cups are in 9 cups?</td>
<td>( 9 \div \frac{1}{3} )</td>
<td>( 9 \times 3 = ? )</td>
<td></td>
</tr>
<tr>
<td>4. How many ( \frac{1}{8} ) pizzas are in 4 pizzas?</td>
<td>( 4 \div \frac{1}{8} )</td>
<td>( 4 \times 8 = ? )</td>
<td></td>
</tr>
<tr>
<td>5. How many one-fifths are in 7 wholes?</td>
<td>( 7 \div \frac{1}{5} )</td>
<td>( 7 \times 5 = ? )</td>
<td></td>
</tr>
</tbody>
</table>

Example 2 (5 minutes)

- All of the problems in the first example show what is called measurement division. When we know the original amount and the size or measure of one part, we use measurement division to find the number of parts. You can tell when a question is asking for measurement division because it asks, “How many ______ are in ____________?”
Let’s take a look at a different example:

Example 2
Molly uses 9 cups of flour to bake bread. If this is \(\frac{3}{4}\) of the total amount of flour she started with, what was the original amount of flour?

How is this question different from the measurement questions?
- In this example, we are not trying to figure out how many three-fourths are in 9. We know that 9 cups is a part of the entire amount of flour needed. Instead, we need to determine three-fourths of what number is 9.

a. Create a model to represent what the question is asking.

![Model](image)

b. Explain how you would determine the answer using the model.

To divide 9 by \(\frac{3}{4}\), we divide 9 by 3 to get the amount for each rectangle; then, we multiply by 4 because there are 4 rectangles total.

\[9 \div \frac{3}{4} = 3 \times 4 = 12\]. Now, I can see that there were originally 12 cups of flour.

![Model](image)
Exercises 1–5 (15 minutes)

Students will work in pairs or on their own to solve the following questions. First, students will write a division expression to represent the situations. Then, students will rewrite each problem as a multiplication question. Finally, they will draw a model to represent the solution.

Allow time for students to share their models. Take time to have students compare the different models that were used to solve each question. For example, allow students to see how a fraction bar and a number line can be used to model Exercise 1.

### Exercises 1–5

1. A construction company is setting up signs on 4 miles of the road. If the company places a sign every $\frac{1}{8}$ of a mile, how many signs will it need?

   $4 \div \frac{1}{8} \rightarrow \frac{1}{8}$ of what number is 4?

   The company will need 32 signs.

2. George bought 12 pizzas for a birthday party. If each person will eat $\frac{3}{8}$ of a pizza, how many people can George feed with 12 pizzas?

   $12 \div \frac{3}{8} \rightarrow \frac{3}{8}$ of what number is 12?

   The pizzas will feed 32 people.

3. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up $\frac{3}{4}$ of a mile, how many family members are needed to clean the adopted section?

   $6 \div \frac{3}{4} \rightarrow \frac{3}{4}$ of what number is 6?

   The Lopez family needs to bring 8 family members to clean the adopted section.
4. Margo is freezing 8 cups of strawberries. If this is \( \frac{2}{3} \) of the total strawberries that were picked, how many cups of strawberries did Margo pick?

\[
8 \div \frac{2}{3} \Rightarrow \frac{2}{3} \text{ of what number is } 8?
\]

\[
\begin{array}{cccc}
4 & 4 & 4 & \quad 8 \\
? & = & 12
\end{array}
\]

*Margo picked 12 cups of strawberries.*

5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent \( \frac{5}{8} \) of all the logs that need to be chopped, how many logs need to be chopped in all?

\[
10 \div \frac{5}{8} \Rightarrow \frac{5}{8} \text{ of what number is } 10?
\]

\[
\begin{array}{ccccccc}
2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\
? & & & & & 10
\end{array}
\]

*Regina needs to chop 16 logs in all.*

**Closing (5 minutes)**

- What are the key ideas from Lessons 1 and 2?
  - *We can use models to divide a whole number by a fraction and a fraction by a whole number.*
  - Over the past two lessons, we have reviewed how to divide a whole number by a fraction and how to divide a fraction by a whole number. The next two lessons will focus on dividing fractions by fractions. Explain how you would use what we have learned about dividing with fractions in the next two lessons.
  - *We can use models to help us divide a fraction by a fraction. We can also use the multiplication problems we wrote as a tool to help us divide fractions by fractions.*

**Exit Ticket (5 minutes)**
Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Exit Ticket

Solve each division problem using a model.

1. Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

2. Rachel completed $\frac{3}{4}$ of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?
Exit Ticket Sample Solutions

Solve each division problem using a model.

1. Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

   \[ 4 \div \frac{1}{6} \] of what is 4?

   \[ \begin{array}{cccccc}
   & & & & & \\
   & & & & & 4 \\
   & & & & & 8 \\
   & & & & & 12 \\
   & & & & & 16 \\
   & & & & & 20 \\
   & & & & & 24 \\
   \end{array} \]

   24 people are in the group.

2. Rachel completed \( \frac{3}{4} \) of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?

   \[ 6 \div \frac{3}{4} \]

   \[ \frac{3}{4} \] of what is 6?

   \[ \begin{array}{cccc}
   & & & \\
   2 & 2 & 2 & \\
   \end{array} \]

   Rachel will spend 8 total hours cleaning.

Problem Set Sample Solutions

Rewrite each problem as a multiplication question. Model your answer.

1. Nicole has used 6 feet of ribbon. This represents \( \frac{3}{8} \) of the total amount of ribbon she started with. How much ribbon did Nicole have at the start?

   \[ 6 \div \frac{3}{8} \]

   \[ \frac{3}{8} \] of what number is 6?

   \[ \begin{array}{ccccccc}
   & & & & & & \\
   & & & & & & 2 \\
   & & & & & & 2 \\
   & & & & & & 2 \\
   & & & & & & 2 \\
   & & & & & & 2 \\
   & & & & & & 2 \\
   \end{array} \]

   Nicole started with 16 feet of ribbon.
2. How many quarter hours are in 5 hours?

\[
5 \div \frac{1}{4}
\]

\[
\frac{1}{4} \text{ of what is } 5?
\]

| 5 | 5 | 5 | 5 |

There are 20 quarter hours in 5 hours.
Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Classwork

Example 1

Question #_______

Write it as a division question. ________________________________

Write it as a multiplication question. ________________________________

Make a rough draft of a model to represent the question:
As you travel to each model, be sure to answer the following questions:

<table>
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<tr>
<th>Original Questions</th>
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<th>What multiplication question could the model also answer?</th>
<th>Write the question given to each group as a multiplication question.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How many $\frac{1}{2}$ miles are in 12 miles?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. How many quarter hours are in 5 hours?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. How many $\frac{1}{3}$ cups are in 9 cups?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. How many $\frac{1}{8}$ pizzas are in 4 pizzas?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. How many one-fifths are in 7 wholes?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 2

Molly uses 9 cups of flour to bake bread. If this is $\frac{3}{4}$ of the total amount of flour she started with, what was the original amount of flour?

a. Create a model to represent what the question is asking.

b. Explain how you would determine the answer using the model.

Exercises 1–5

1. A construction company is setting up signs on 4 miles of the road. If the company places a sign every $\frac{1}{8}$ of a mile, how many signs will it need?
2. George bought 12 pizzas for a birthday party. If each person will eat $\frac{3}{8}$ of a pizza, how many people can George feed with 12 pizzas?

3. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up $\frac{3}{4}$ of a mile, how many family members are needed to clean the adopted section?
4. Margo is freezing 8 cups of strawberries. If this is \( \frac{2}{3} \) of the total strawberries that were picked, how many cups of strawberries did Margo pick?

5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent \( \frac{5}{8} \) of all the logs that need to be chopped, how many logs need to be chopped in all?
Problem Set

Rewrite each problem as a multiplication question. Model your answer.

1. Nicole has used 6 feet of ribbon. This represents \( \frac{3}{8} \) of the total amount of ribbon she started with. How much ribbon did Nicole have at the start?

2. How many quarter hours are in 5 hours?