These 5 lessons do a very good job with nets and surface area. If you like these lessons, please consider using other Eureka lessons as well.

Lesson 15

- Page 2-3  Informational  *(page 2’s diagram is great)*
- Pages 4-8  Teacher Pages  *Representing 3D Figures with Nets*
- Pages 9-11  Exit Ticket w/ solutions for Lesson 15
- Pages 12-36  Various nets - *See my note on page 5 about altering activity.*
- Pages 37-39  Students pages for Lesson 15

Lesson 16

- Pages 40-45  Teacher Pages  *Constructing Nets*
- Pages 46-49  Exit Ticket w/ solutions for Lesson 16
- Pages 50-55 – *rectangles & triangles for exercises 1-3*
- Pages 56-59 – Student pages for Lesson 16

Lesson 17

- Pages 60-66  Teacher Pages  *From Nets to Surface Area*
- Pages 67-70  Exit Ticket w/ solutions for Lesson 17
- Page 71-76  Student pages for Lesson 17

Lesson 18

- Pages 77-82  Teacher Pages  *Volume in the Real World*
- Pages 83-86  Exit Ticket w/ solutions for Lesson 13
- Page 87-93  Student pages for Lesson 13

Lesson 19

- Pages 94-97  Teacher Pages for  *Surface Area & Volume in the Real World*
- Pages 98-105  Exit Ticket & solutions
- Pages 106-110  Student Pages for Lesson 19
Topic D: Nets and Surface Area


**Focus Standard:**

- **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

- **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

**Instructional Days:** 5

- **Lesson 15:** Representing Three-Dimensional Figures Using Nets (M)
- **Lesson 16:** Constructing Nets (E)
- **Lesson 17:** From Nets to Surface Area (P)
- **Lesson 18:** Determining Surface Area of Three-Dimensional Figures (P)
- **Lesson 19:** Surface Area and Volume in the Real World (P)
- **Lesson 19a:** Addendum Lesson for Modeling—Applying Surface Area and Volume to Aquariums (Optional) (M)

Topic D begins with students constructing three-dimensional figures through the use of nets in Lesson 15. They determine which nets make specific solid figures and also determine if nets can or cannot make a solid figure. Students use physical models and manipulatives to do actual constructions of three-dimensional figures with the nets. Then, in Lesson 16, students move to constructing nets of three-dimensional objects using the measurements of a solid’s edges. Using this information, students will move from nets to determining the surface area of three-dimensional figures in Lesson 17. In Lesson 18, students determine that a right rectangular prism has six faces: top and bottom, front and back, and two sides. They determine

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1 Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

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that surface area is obtained by adding the areas of all the faces and develop the formula
\[ SA = 2lw + 2lh + 2wh. \]
They develop and apply the formula for the surface area of a cube as \( SA = 6s^2 \).

For example:

![Diagram of a cube with dimensions labeled](image)

<table>
<thead>
<tr>
<th>Top</th>
<th>Bottom</th>
<th>Front</th>
<th>Back</th>
<th>Side</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l \times w )</td>
<td>( l \times w )</td>
<td>( l \times h )</td>
<td>( l \times h )</td>
<td>( w \times h )</td>
<td>( w \times h )</td>
</tr>
<tr>
<td>8 cm \times 2 cm</td>
<td>8 cm \times 2 cm</td>
<td>8 cm \times 3 cm</td>
<td>8 cm \times 3 cm</td>
<td>2 cm \times 3 cm</td>
<td>2 cm \times 3 cm</td>
</tr>
<tr>
<td>16 cm(^2)</td>
<td>16 cm(^2)</td>
<td>24 cm(^2)</td>
<td>24 cm(^2)</td>
<td>6 cm(^2)</td>
<td>6 cm(^2)</td>
</tr>
</tbody>
</table>

\[ SA = 16 \text{ cm}^2 + 16 \text{ cm}^2 + 24 \text{ cm}^2 + 24 \text{ cm}^2 + 6 \text{ cm}^2 + 6 \text{ cm}^2 = 92 \text{ cm}^2 \]

\[ l \times w \quad l \times w \quad l \times h \quad l \times h \quad w \times h \quad w \times h \]

\[ 2lw + 2lh + 2wh \]

Topic D concludes with Lesson 19, in which students determine the surface area of three-dimensional figures in real-world contexts. To develop skills related to application, students are exposed to contexts that involve both surface area and volume. Students are required to make sense of each context and apply concepts appropriately.
Lesson 15: Representing Three-Dimensional Figures Using Nets

Student Outcomes

- Students construct three-dimensional figures through the use of nets. They determine which nets make specific solid figures and determine if nets can or cannot make a solid figure.

Lesson Notes

Using geometric nets is a topic that has layers of sequential understanding as students progress through the years. For Grade 6, specifically in this lesson, the working description of a net is this: If the surface of a three-dimensional solid can be cut along enough edges so that the faces can be placed in one plane to form a connected figure, then the resulting system of faces is called a net of the solid.

A more student-friendly description used for this lesson is the following: Nets are two-dimensional figures that can be folded to create three-dimensional solids.

Solid figures and the nets that represent them are necessary for this lesson. These three-dimensional figures include a cube, a right rectangular prism, a triangular prism, a tetrahedron, a triangular pyramid (equilateral base and isosceles triangular sides), and a square pyramid.

There are reproducible copies of these nets included with this lesson. The nets of the cube and right rectangular prism are sized to wrap around solid figures made from wooden or plastic cubes with 2 cm-edges. Assemble these two solids prior to the lesson in enough quantities to allow students to work in pairs. If possible, the nets should be reproduced on card stock and pre-cut and pre-folded before the lesson. One folded and taped example of each should also be assembled before the lesson.

The triangular prism has a length of 6 cm and has isosceles right triangular bases with identical legs that are 2 cm in length. Two of these triangular prisms can be arranged to form a rectangular prism.

The rectangular prism measures 4 cm × 6 cm × 8 cm, and its net will wrap around a Unifix cube solid that has dimensions of 2 × 3 × 4 cubes.

The tetrahedron has an edge length of 6 cm. The triangular pyramid has a base edge length of 6 cm and isosceles sides with a height of 4 cm.

The square pyramid has a base length 6 cm and triangular faces that have a height of 4 cm.

Also included is a reproducible sheet that contains 20 unique arrangements of six squares. Eleven of these can be folded to a cube, while nine cannot. These should also be prepared before the lesson, as indicated above. Make enough sets of nets to accommodate the number of groups of students.

Prior to the lesson, cut a large cereal box into its net which will be used for the Opening Exercise. Tape the top flaps thoroughly so this net will last through several lessons. If possible, get two identical boxes and cut two different nets like the graphic patterns of the cube nets below. Add a third uncut box to serve as a right rectangular solid model.
Lesson 15

Mathematical Modeling Exercise (10 minutes)

Display the net of the cereal box with the unprinted side out, perhaps using magnets on a whiteboard. Display the nets below as well (images or physical nets).

- What can you say about this cardboard (the cereal box)?
  - Accept all correct answers, such as it is irregularly shaped; it has three sets of identical rectangles; all vertices are right angles; it has fold lines; it looks like it can be folded into a 3D shape (box), etc.

- How do you think it was made?
  - Accept all plausible answers, including the correct one.

- Compare the cereal box net to these others that are made of squares.
  - Similarities: There are 6 sections in each; they can be folded to make a 3D shape; etc.
  - Differences: One is made of rectangles; others are made of squares; there is a size difference; etc.

Turn over the cereal box to demonstrate how it was cut. Reassemble it to resemble the intact box. Then, direct attention to the six-square arrangements.

- What do you think the six-square shapes will fold up into?
  - Cubes

- If that were true, how many faces would it have?
  - Six

Fold each into a cube.

- Consider this six-square arrangement:

- Do you think it will fold into a cube?

Encourage a short discussion, inviting all views. As students make claims, ask for supporting evidence of their position. Use the cut-out version to demonstrate that this arrangement will not fold into a cube. Then, define the term net.

- Today we will work with some two-dimensional figures that can be folded to create three-dimensional solids. These are called geometric nets, or nets.

Ask students if they are able to visualize folding the nets without touching them. Expect a wide variety of spatial visualization abilities necessary to do this. Those that cannot readily see the outcome of folding will need additional time to handle and actually fold the models.
Lesson 15: Representing Three-Dimensional Figures Using Nets

A STORY OF RATIOS

Gail’s Note: This requires 22 sheets of paper per group and the time to cut them all out. Perhaps allow groups to do 3 or 4 nets each and then bring findings to the big group?

Exercise (10 minutes): Cube

Use the previously cut out six-square arrangements. Each pair or triad of students will need a set of 20 with which to experiment. These are sized to wrap around a cube with side lengths of 4 cm, which can be made from eight Unifix cubes. Each group needs one of these cubes.

- There are some six-square arrangements on your student page. Sort each of the six-square arrangements into one of two piles, those that are nets of a cube (can be folded into a cube) and those that are not.

Exercise: Cube

1. Nets are two-dimensional figures that can be folded up into three-dimensional solids. Some of the drawings below are nets of a cube. Others are not cube nets; they can be folded, but not into a cube.


   a. Experiment with the larger cut out patterns provided. Shade in each of the figures above that will fold into a cube.

   b. Write the letters of the figures that can be folded up into a cube.

   \[A, B, C, E, G, I, L, M, O, P, \text{ and } T\]

   c. Write the letters of the figures that cannot be folded up into a cube.

   \[D, F, H, J, K, N, Q, R, \text{ and } S\]

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Example 1 (10 minutes): Other Solid Figures

Provide student pairs with a set of nets for each of the following: right rectangular prism, triangular prism, tetrahedron, triangular pyramid (equilateral base and isosceles triangular sides), and square pyramid.

PRISM: A prism is a solid geometric figure whose two bases are parallel to identical polygons and whose sides are parallelograms.

PYRAMID: A pyramid is a solid geometric figure formed by connecting a polygonal base and a point and forming triangular lateral faces. (Note: The point is sometimes referred to as the apex.)

Display one of each solid figure. Assemble them so the grid lines are hidden (inside).

Allow time to explore the nets folding around the solids.

- Why are the faces of the pyramid triangles?
  - The base of the triangle matches the edge of the base of the pyramid. The top vertex of the lateral face is at the apex of the pyramid. Further, each face has two vertices that are the endpoints of one edge of the pyramid’s base, and the third vertex is the apex of the pyramid.

- Why are the faces of the prism parallelograms?
  - The two bases are identical polygons on parallel planes. The lateral faces are created by connecting each vertex of one base with the corresponding vertex of the other base, thus forming parallelograms.

- How are these parallelograms related to the shape and size of the base?
  - The length of the base edges will match one set of sides of the parallelogram. The shape of the base polygon will determine the number of lateral faces the prism has.

- If the bases are hexagons, does this mean the prism must have six faces?
  - No, there are six sides on the prism, plus two bases, for a total of eight faces.

- What is the relationship between the number of sides on the polygonal base and the number of faces on the prism?
  - The total number of faces will be two more than the number of sides on the polygonal bases.

- What additional information do you know about a prism if its base is a regular polygon?
  - All the lateral faces of the prism will be identical.

Example 2 (8 minutes): Tracing Nets

If time allows, or as an extension, ask students to trace the faces of various solid objects (i.e., wooden or plastic geosolids, paperback books, packs of sticky notes, or boxes of playing cards). After tracing a face, the object should be carefully rolled so one edge of the solid matches one side of the polygon that has just been traced. If this is difficult for students because they lose track of which face is which as they are rolling, the faces can be numbered or colored differently to make this easier. These drawings should be labeled “Net of a [Name of Solid].” Challenge students to make as many different nets of each solid as they can.

Scaffolding:
- Assembled nets of each solid figure should be made available to students who might have difficulty making sharp, precise folds.
- All students may benefit from a working definition of the word lateral. In this lesson, the word side can be used (as opposed to the word base).
- English language learners may hear similarities to the words ladder or literal, neither of which are related nor make sense in this context.
Closing (3 minutes)

- What kind of information can be obtained from a net of a prism about the solid it creates?
  - We can identify the shape of the bases and the number and shape of the lateral faces (sides). The surface area can be more easily obtained since we can see all faces at once.

- When looking at a net of a pyramid, how can you determine which faces are the bases?
  - If the net is a pyramid, there will be multiple, identical triangles that will form the lateral faces of the pyramid, while the remaining face will be the base (and will identify the type of pyramid it is). Examples are triangular, square, pentagonal, and hexagonal pyramids.

- How do the nets of a prism differ from the nets of a pyramid?
  - If the pyramid is not a triangular pyramid, the base will be the only polygon that is not a triangle. All other faces will be triangles. Pyramids have one base and triangular lateral faces, while prisms have two identical bases, which could be any type of polygon, and lateral faces that are parallelograms.

- Constructing solid figures from their nets helps us see the “suit” that fits around it. We can use this in our next lesson to find the surface area of these solid figures as we wrap them.

Lesson Summary

Nets are two-dimensional figures that can be folded to create three-dimensional solids.

A prism is a solid geometric figure whose two bases are parallel to identical polygons and whose sides are parallelograms.

A pyramid is a solid geometric figure formed by connecting a polygonal base and a point and forming triangular lateral faces. (Note: The point is sometimes referred to as the apex.)

Exit Ticket (4 minutes)
Lesson 15: Representing Three-Dimensional Figures Using Nets

Exit Ticket

1. What is a net? Describe it in your own words.

2. Which of the following will fold to make a cube? Explain how you know.
Exit Ticket Sample Solutions

1. What is a net? Describe it in your own words.

   Answers will vary but should capture the essence of the definition used in this lesson. A net is a two-dimensional figure that can be folded to create a three-dimensional solid.

2. Which of the following will fold to make a cube? Explain how you know.

   Evidence for claims will vary.

Problem Set Sample Solutions

1. Match the following nets to the picture of its solid. Then, write the name of the solid.

   a. 
   d. Right triangular prism

   b. 
   e. Rectangular pyramid

   c. 
   f. Rectangular prism
2. Sketch a net that will fold into a cube.
   *Sketches will vary but will match one of the shaded ones from earlier in the lesson.*

   *Here are the 11 possible nets for a cube.*

3. Below are the nets for a variety of prisms and pyramids. Classify the solids as prisms or pyramids, and identify the shape of the base(s). Then, write the name of the solid.

   a. 
   ![Net Image]
   
   **Prism, the bases are pentagons.**
   **Pentagonal Prism**

   b. 
   ![Net Image]
   
   **Pyramid, the base is a rectangle.**
   **Rectangular Pyramid**

   c. 
   ![Net Image]
   
   **Pyramid, the base is a triangle.**
   **Triangular Pyramid**

   d. 
   ![Net Image]
   
   **Prism, the bases are triangles.**
   **Triangular Prism**

   e. 
   ![Net Image]
   
   **Pyramid, the base is a hexagon.**
   **Hexagonal Pyramid**

   f. 
   ![Net Image]
   
   **Prism, the bases are rectangles.**
   **Rectangular Prism**

Below are graphics needed for this lesson. The graphics should be printed at 100% scale to preserve the intended size of figures for accurate measurements. Adjust your copier or printer settings to actual size, and set page scale to none.
Lesson 15: Representing Three-Dimensional Figures Using Nets
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Lesson 15: Representing Three-Dimensional Figures Using Nets
Part 1 of 2
Part 2 of 2
Lesson 15:
Representing Three-Dimensional Figures Using Nets
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Lesson 15: Representing Three-Dimensional Figures Using Nets

Classwork

Exercise: Cube

1. Nets are two-dimensional figures that can be folded up into three-dimensional solids. Some of the drawings below are nets of a cube. Others are not cube nets; they can be folded, but not into a cube.

   \[ \text{A} \quad \text{B} \quad \text{C} \quad \text{D} \quad \text{E} \]
   \[ \text{F} \quad \text{G} \quad \text{H} \quad \text{I} \quad \text{J} \]
   \[ \text{K} \quad \text{L} \quad \text{M} \quad \text{N} \quad \text{O} \]
   \[ \text{P} \quad \text{Q} \quad \text{R} \quad \text{S} \quad \text{T} \]

   a. Experiment with the larger cut out patterns provided. Shade in each of the figures above that will fold into a cube.

   b. Write the letters of the figures that can be folded up into a cube.

   c. Write the letters of the figures that cannot be folded up into a cube.
Lesson Summary

Nets are two-dimensional figures that can be folded to create three-dimensional solids.

A prism is a solid geometric figure whose two bases are parallel to identical polygons and whose sides are parallelograms.

A pyramid is a solid geometric figure formed by connecting a polygonal base and a point and forming triangular lateral faces. (Note: The point is sometimes referred to as the apex.)

Problem Set

1. Match the following nets to the picture of its solid. Then, write the name of the solid.

   a. 
   d. 

   b. 
   e. 

   c. 
   f.
2. Sketch a net that will fold into a cube.

3. Below are the nets for a variety of prisms and pyramids. Classify the solids as prisms or pyramids, and identify the shape of the base(s). Then, write the name of the solid.

   a.
   b.
   c.
   d.
   e.
   f.
Lesson 16: Constructing Nets

Student Outcomes
- Students construct nets of three-dimensional objects using the measurements of a solid’s edges.

Lesson Notes
In the previous lesson, a cereal box was cut down to one of its nets. On the unprinted side, the fold lines should be highlighted with a thick marker to make all six faces easily seen. These rectangles should be labeled Front, Back, Top, Bottom, Left Side, and Right Side. Measure each rectangle to the nearest inch, and record the dimensions on each.

During this lesson, students are given the length, width, and height of a right rectangular solid. They cut out six rectangles (three pairs), arrange them into a net, tape them, and fold them up to check the arrangement to ensure the net makes the solid. Triangular pieces are also used in constructing the nets of pyramids and triangular prisms.

When students construct the nets of rectangular prisms, if no two dimensions, length, width, or height, are equal, then no two adjacent rectangular faces will be identical.

The nets that were used in Lesson 15 should be available so that students have the general pattern layout of the nets.

Two-centimeter graph paper works well with this lesson. Prior to the lesson, cut enough polygons for Example 1. Cutting all the nets used in this lesson will save time as well but removes the opportunity for students to do the work.

Classwork

Opening (2 minutes)
Display the cereal box net from the previous lesson. Fold and unfold it so students will recall the outcome of the lesson.

- How has this net changed since the previous lesson?
  - It now has labels and dimensions.
- What can you say about the angles in each rectangle?
  - They are 90 degrees, or right angles.
- What can you say about the angles between the faces when it is folded up?
  - The two faces also form a right angle.
- What can you say about the vertices where 3 faces come together?
  - Again, they form right angles.
- This refolded box is an example of a right rectangular prism. It is named for the angles formed at each vertex.

Scaffolding:
Some students will need more opportunities than others to manipulate the nets in this lesson.
Opening Exercise (3 minutes)

Sketch the faces in the area below. Label the dimensions.

Display this graphic using a document camera or other device.

- How could you create a net for this solid? Discuss this with a partner.

Allow a short time for discussion with a partner about this before having a whole-class discussion.

Example 1 (10 minutes): Right Rectangular Prism

- How can we use the dimensions of a rectangular solid to figure out the dimensions of the polygons that make up its net?
  - The length, width, and height measurements of the solid will be paired to become the length and width of the rectangles.
- How many faces does the rectangular prism have?
  - 6
- What are the dimensions of the top of this prism?
  - 8 cm × 3 cm
- What are the dimensions of the bottom?
  - 8 cm × 3 cm
- What are the dimensions of the right side?
  - 3 cm × 5 cm
- What are the dimensions of the left side?
  - 3 cm × 5 cm
- What are the dimensions of the front?
  - 8 cm × 5 cm
- What are the dimensions of the back?
  - 8 cm × 5 cm
- The 6 faces of this rectangular solid are all rectangles that make up the net. Are there any faces that are identical to any others?
  - There are three different rectangles, but two copies of each will be needed to make the solid. The top is identical to the bottom, the left and right sides are identical, and the front and back faces are also identical.

Make sure each student can visualize the rectangles depicted on the graphic of the solid and can make three different pairs of rectangle dimensions (length × width, length × height, and width × height).

Display the previously cut six rectangles from this example on either an interactive whiteboard or on a magnetic surface. Discuss the arrangement of these rectangles. Identical sides must match.

Working in pairs, ask students to rearrange the rectangles into the shape below and use tape to attach them. Having a second copy of these already taped will save time during the lesson.

Scaffolding:
- Some students will benefit from using precut rectangles and triangles. Using cardstock or lamination will make more durable polygons.
- Other students benefit from tracing the faces of actual solids onto paper and then cutting and arranging them.
• If this is truly a net of the solid, it will fold up into a box. In mathematical language, it is known as a right rectangular prism.

Students should fold the net into the solid to prove that it was indeed a net. Be prepared for questions about other arrangements of these rectangles that are also nets of the right rectangular prism. There are many possible arrangements.

Exploratory Challenge 1 (9 minutes): Rectangular Prisms

Students will make nets from given measurements. Rectangles should be cut from graph paper and taped. Ask students to have their rectangle arrangements checked before taping. After taping, it can be folded to check its fidelity.

Exploratory Challenge 1: Rectangular Prisms

a. Use the measurements from the solid figures to cut and arrange the faces into a net.

One possible configuration of rectangles is shown here:

b. A juice box measures 4 inches high, 3 inches long, and 2 inches wide. Cut and arrange all 6 faces into a net.

One possible configuration of faces is shown here:
c.  Challenge: Write a numerical expression for the total area of the net for part (b). Explain each term in your expression.

Possible answer: \(2(2 \text{ in.} \times 3 \text{ in.}) + 2(2 \text{ in.} \times 4 \text{ in.}) + 2(3 \text{ in.} \times 4 \text{ in.})\). There are two sides that have dimensions 2 in. by 3 in., two sides that are 2 in. by 4 in., and two sides that are 3 in. by 4 in.

Exploratory Challenge 2 (7 minutes): Triangular Prisms

Cutting these prior to the lesson will save time during the lesson.

Exploratory Challenge 2: Triangular Prisms
Use the measurements from the triangular prism to cut and arrange the faces into a net.

One possible configuration of rectangles and triangles is shown here:

Exploratory Challenge 3 (9 minutes): Pyramids

Exploratory Challenge 3: Pyramids
Pyramids are named for the shape of the base.

a.  Use the measurements from this square pyramid to cut and arrange the faces into a net. Test your net to be sure it folds into a square pyramid.

One possible configuration of rectangles and triangles is shown here:
Lesson 16: Constructing Nets

b. A triangular pyramid that has equilateral triangles for faces is called a tetrahedron. Use the measurements from this tetrahedron to cut and arrange the faces into a net. All edges measure 4 inches.

One possible configuration of triangles is shown here:

Closing (2 minutes)

- What are the most important considerations when making nets of solid figures?
  - Each face must be taken into account.
- After all faces are made into polygons (either real or drawings), what can you say about the arrangement of those polygons?
  - Edges must match like on the solid.
- Describe the similarities between the nets of right rectangular prisms.
  - All faces are rectangles. Opposite faces are identical rectangles. If the base is a square, the lateral faces are identical rectangles. If the prism is a cube, all of the faces are identical.
- Describe the similarities between the nets of pyramids.
  - All of the faces that are not the base are triangles. The number of these faces is equal to the number of sides the base contains. If the base is a regular polygon, the faces are identical triangles. If all of the faces of a triangular pyramid are identical, then the solid is a tetrahedron.
- How can you test your net to be sure that it is really a true net of the solid?
  - Make a physical model and fold it up.

Exit Ticket (3 minutes)
Lesson 16: Constructing Nets

Exit Ticket

Sketch and label a net of this pizza box. It has a square top that measures 16 inches on a side, and the height is 2 inches. Treat the box as a prism, without counting the interior flaps that a pizza box usually has.
Exit Ticket Sample Solutions

Sketch and label a net of this pizza box. It has a square top that measures 16 inches on a side, and the height is 2 inches. Treat the box as a prism, without counting the interior flaps that a pizza box usually has.

One possible configuration of faces is shown here:

Problem Set Sample Solutions

1. Sketch and label the net of the following solid figures, and label the edge lengths.
   a. A cereal box that measures 13 inches high, 7 inches long, and 2 inches wide

   One possible configuration of faces is shown here:
Lesson 16: Constructing Nets

b. A cubic gift box that measures 8 cm on each edge

*One possible configuration of faces is shown here:*

All edges are 8 cm.

![Net of a cube]

There are 6 faces in the cube, and each has dimensions 8 cm by 8 cm.

Challenge: Write a numerical expression for the total area of the net in part (b). Tell what each of the terms in your expression means.

\[6(8 \text{ cm} \times 8 \text{ cm}) \text{ or } (8 \text{ cm} \times 8 \text{ cm}) + (8 \text{ cm} \times 8 \text{ cm}) + (8 \text{ cm} \times 8 \text{ cm}) + (8 \text{ cm} \times 8 \text{ cm}) + (8 \text{ cm} \times 8 \text{ cm}) + (8 \text{ cm} \times 8 \text{ cm})\]

2. This tent is shaped like a triangular prism. It has equilateral bases that measure 5 feet on each side. The tent is 8 feet long. Sketch the net of the tent, and label the edge lengths.

*Possible net:*

![Net of a triangular prism]
3. The base of a table is shaped like a square pyramid. The pyramid has equilateral faces that measure 25 inches on each side. The base is 25 inches long. Sketch the net of the table base, and label the edge lengths.

*Possible net:*

![Net of a Square Pyramid](image)

4. The roof of a shed is in the shape of a triangular prism. It has equilateral bases that measure 3 feet on each side. The length of the roof is 10 feet. Sketch the net of the roof, and label the edge lengths.

*Possible net:*

![Net of a Triangular Prism](image)
Rectangles for Opening Exercise
Rectangles for Exercise 1, part (a)
Rectangles for Exercise 1, part (b)

- 4 in.
- 3 in.

- 4 in.
- 2 in.

- 3 in.
- 2 in.
Polygons for Exercise 2

3 in.  3 in.  3 in.  3 in.

3 in.  3 in.

6 in.

3 in.
Polygons for Exercise 3, part (a)
Triangles for Exercise 3, part (b)
Lesson 16: Constructing Nets

Classwork

Opening Exercise

Sketch the faces in the area below. Label the dimensions.

![Diagram of a 3D shape with dimensions 3, 5, and 8]
Exploratory Challenge 1: Rectangular Prisms

a. Use the measurements from the solid figures to cut and arrange the faces into a net.

b. A juice box measures 4 inches high, 3 inches long, and 2 inches wide. Cut and arrange all 6 faces into a net.

c. Challenge: Write a numerical expression for the total area of the net for part (b). Explain each term in your expression.

Exploratory Challenge 2: Triangular Prism

Use the measurements from the triangular prism to cut and arrange the faces into a net.
Exploratory Challenge 3: Pyramids

Pyramids are named for the shape of the base.

a. Use the measurements from this square pyramid to cut and arrange the faces into a net. Test your net to be sure it folds into a square pyramid.

b. A triangular pyramid that has equilateral triangles for faces is called a tetrahedron. Use the measurements from this tetrahedron to cut and arrange the faces into a net.
Problem Set

1. Sketch and label the net of the following solid figures, and label the edge lengths.
   a. A cereal box that measures 13 inches high, 7 inches long, and 2 inches wide
   b. A cubic gift box that measures 8 cm on each edge
   c. Challenge: Write a numerical expression for the total area of the net in part (b). Tell what each of the terms in your expression means.

2. This tent is shaped like a triangular prism. It has equilateral bases that measure 5 feet on each side. The tent is 8 feet long. Sketch the net of the tent, and label the edge lengths.

3. The base of a table is shaped like a square pyramid. The pyramid has equilateral faces that measure 25 inches on each side. The base is 25 inches long. Sketch the net of the table base, and label the edge lengths.

4. The roof of a shed is in the shape of a triangular prism. It has equilateral bases that measure 3 feet on each side. The length of the roof is 10 feet. Sketch the net of the roof, and label the edge lengths.
Lesson 17: From Nets to Surface Area

Student Outcomes
- Students use nets to determine the surface area of three-dimensional figures.

Classwork

Fluency Exercise (5 minutes): Addition and Subtraction Equations

Sprint: Refer to the Sprints and the Sprint Delivery Script sections of the Module Overview for directions to administer a Sprint.

Opening Exercise (4 minutes)

Students work independently to calculate the area of the shapes below.

Opening Exercise

a. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.
   i. \[ A = \frac{1}{2}(14 \text{ cm})(12 \text{ cm}) = 84 \text{ cm}^2 \]
   14 cm represents the base of the figure because 
   5 cm + 9 cm = 14 cm, and 12 cm represents the altitude of the figure because it forms a right angle with the base.

   ii. How would you write an equation that shows the area of a triangle with base \( b \) and height \( h \)?
   \[ A = \frac{1}{2}bh \]

b. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.
   i. \[ A = (28 \text{ ft.})(18 \text{ ft.}) = 504 \text{ ft}^2 \]
   28 ft. represents the base of the rectangle, and 18 ft. represents the height of the rectangle.

   ii. How would you write an equation that shows the area of a rectangle with base \( b \) and height \( h \)?
   \[ A = bh \]
Discussion (5 minutes)

English language learners may not recognize the word *surface*; take this time to explain what *surface area* means. Demonstrate that *surface* is the upper or outer part of something, like the top of a desk. Therefore, *surface area* is the area of all the faces, including the bases of a three-dimensional figure.

Use the diagram below to discuss nets and surface area.

- Examine the net on the left and the three-dimensional figure on the right. What do you notice about the two diagrams?
  - *The two diagrams represent the same rectangular prism.*

- Examine the second rectangular prism in the center column. The one shaded face is the back of the figure, which matches the face labeled *back* on the net. What do you notice about those two faces?
  - *The faces are identical and will have the same area.*

Continue the discussion by talking about one rectangular prism pictured at a time, connecting the newly shaded face with the identical face on the net.

- Will the surface area of the net be the same as the surface area of the rectangular prism? Why or why not?
  - *The surface area for the net and the rectangular prism will be the same because all the matching faces are identical, which means their areas are also the same.*
Example 1 (4 minutes)

Lead students through the problem.

Example 1
Use the net to calculate the surface area of the figure.

- When you are calculating the area of a figure, what are you finding?
  - The area of a figure is the amount of space inside a two-dimensional figure.
- Surface area is similar to area, but surface area is used to describe three-dimensional figures. What do you think is meant by the surface area of a solid?
  - The surface area of a three-dimensional figure is the area of each face added together.
- What type of figure does the net create? How do you know?
  - It creates a rectangular prism because there are six rectangular faces.
- If the boxes on the grid paper represent a 1 cm × 1 cm box, label the dimensions of the net.
The surface area of a figure is the sum of the areas of all faces. Calculate the area of each face, and record this value inside the corresponding rectangle.

- In order to calculate the surface area, we will have to find the sum of the areas we calculated since they represent the area of each face. There are two faces that have an area of 4 cm² and four faces that have an area of 2 cm². How can we use these areas to write a numerical expression to show how to calculate the surface area of the net?
  - The numerical expression to calculate the surface area of the net would be:
    
    $$\text{(1 cm × 2 cm)} + \text{(1 cm × 2 cm)} + \text{(1 cm × 2 cm)} + \text{(1 cm × 2 cm)} + \text{(2 cm × 2 cm)} + \text{(2 cm × 2 cm)}.$$ 

- Write the expression more compactly, and explain what each part represents on the net.
  - The expression means there are 4 rectangles that have dimensions 1 cm × 2 cm on the net and 2 rectangles that have dimensions 2 cm × 2 cm on the net.

- What is the surface area of the net?
  - The surface area of the net is 16 cm².

Example 2 (4 minutes)

Lead students through the problem.

Example 2

Use the net to write an expression for surface area.
What type of figure does the net create? How do you know?
- *It creates a square pyramid because one face is a square and the other four faces are triangles.*

If the boxes on the grid paper represent a 1 ft. × 1 ft. square, label the dimensions of the net.

How many faces does the rectangular pyramid have?
- 5

Knowing the figure has 5 faces, use the knowledge you gained in Example 1 to calculate the surface area of the rectangular pyramid.
- **Area of Base:** 3 ft. × 3 ft. = 9 ft²
- **Area of Triangles:** \( \frac{1}{2} \times 3 \text{ ft.} \times 2 \text{ ft.} = 3 \text{ ft}^2 \)
- **Surface Area:** \( 9 \text{ ft}^2 + 3 \text{ ft}^2 + 3 \text{ ft}^2 + 3 \text{ ft}^2 + 3 \text{ ft}^2 = 21 \text{ ft}^2 \)

**Exercises (13 minutes)**

Students work individually to calculate the surface area of the figures below.

**Exercises**

Name the solid the net would create, and then write an expression for the surface area. Use the expression to determine the surface area. Assume that each box on the grid paper represents a 1 cm × 1 cm square. Explain how the expression represents the figure.

1. **Name of Shape:** Rectangular Pyramid, but more specifically a Square Pyramid
   **Surface Area:** \( 4 \text{ cm} \times 4 \text{ cm} + 4 \left( \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} \right) = 16 \text{ cm}^2 + 4(6 \text{ cm}^2) = 40 \text{ cm}^2 \)
   *The figure is made up of a square base that is 4 cm × 4 cm and four triangles with a base of 4 cm and a height of 3 cm.*
2. Name of Shape: Rectangular Prism
Surface Area: $2(5 \text{ cm} \times 5 \text{ cm}) + 4(5 \text{ cm} \times 2 \text{ cm}) = 2(25 \text{ cm}^2) + 4(10 \text{ cm}^2) = 90 \text{ cm}^2$
The figure has 2 square faces that are $5 \text{ cm} \times 5 \text{ cm}$ and 4 rectangular faces that are $5 \text{ cm} \times 2 \text{ cm}$.

3. Name of Shape: Rectangular Pyramid
Surface Area: $3 \text{ cm} \times 4 \text{ cm} + 2 \left( \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} \right) + 2 \left( \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} \right) = 12 \text{ cm}^2 + 2(8 \text{ cm}^2) + 2(6 \text{ cm}^2) = 40 \text{ cm}^2 = 40 \text{ cm}^2$
The figure has 1 rectangular base that is $3 \text{ cm} \times 4 \text{ cm}$, 2 triangular faces that have a base of $4 \text{ cm}$ and a height of $3 \text{ cm}$, and 2 other triangular faces with a base of $3 \text{ cm}$ and a height of $4 \text{ cm}$.

4. Name of Shape: Rectangular Prism
Surface Area: $2(6 \text{ cm} \times 5 \text{ cm}) + 2(5 \text{ cm} \times 1 \text{ cm}) + 2(6 \text{ cm} \times 1 \text{ cm}) = 2(30 \text{ cm}^2) + 2(5 \text{ cm}^2) + 2(6 \text{ cm}^2) = 2(30 \text{ cm}^2) + 2(5 \text{ cm}^2) + 2(6 \text{ cm}^2)$
The figure has 2 rectangular faces that are $6 \text{ cm} \times 5 \text{ cm}$, 2 rectangular faces that are $5 \text{ cm} \times 1 \text{ cm}$, and the final 2 faces that are $6 \text{ cm} \times 1 \text{ cm}$.
Closing (5 minutes)

- Why is a net helpful when calculating the surface area of pyramids and prisms?
  - Answers will vary. The nets are helpful when calculating surface area because it is easier to find the areas of all the faces.

- What type of pyramids and/or prisms requires the fewest calculations when finding surface area?
  - Regular pyramids or prisms require the fewest calculations because the lateral faces are identical, so the faces have equal areas.

Exit Ticket (5 minutes)
Lesson 17: From Nets to Surface Area

Exit Ticket

Name the shape, and then calculate the surface area of the figure. Assume each box on the grid paper represents a 1 in. × 1 in. square.
Exit Ticket Sample Solutions

Name the shape, and then calculate the surface area of the figure. Assume each box on the grid paper represents a 1 in. × 1 in. square.

**Name of Shape: Rectangular Pyramid**

**Area of Base:** \(5 \text{ in.} \times 4 \text{ in.} = 20 \text{ in}^2\)

**Area of Triangles:**
- \(\frac{1}{2} \times 4 \text{ in.} \times 4 \text{ in.} = 8 \text{ in}^2\)
- \(\frac{1}{2} \times 5 \text{ in.} \times 4 \text{ in.} = 10 \text{ in}^2\)

**Surface Area:** \(20 \text{ in}^2 + 8 \text{ in}^2 + 10 \text{ in}^2 + 10 \text{ in}^2 = 56 \text{ in}^2\)

Problem Set Sample Solutions

Name the shape, and write an expression for surface area. Calculate the surface area of the figure. Assume each box on the grid paper represents a 1 ft. × 1 ft. square.

1. **Name of Shape: Rectangular Prism**

**Surface Area:**
- \((2 \text{ ft.} \times 4 \text{ ft.}) + (2 \text{ ft.} \times 4 \text{ ft.}) + (4 \text{ ft.} \times 7 \text{ ft.}) + (4 \text{ ft.} \times 7 \text{ ft.}) + (7 \text{ ft.} \times 2 \text{ ft.}) + (7 \text{ ft.} \times 2 \text{ ft.})\)
- \(2(2 \text{ ft.} \times 4 \text{ ft.}) + 2(4 \text{ ft.} \times 7 \text{ ft.}) + 2(7 \text{ ft.} \times 2 \text{ ft.})\)
- \(16 \text{ ft}^2 + 56 \text{ ft}^2 + 28 \text{ ft}^2 = 100 \text{ ft}^2\)
2. **Name of Shape: Rectangular Pyramid**

Surface Area: 
\[
(2 \text{ ft} \times 5 \text{ ft}) + \left(\frac{1}{2} \times 2 \text{ ft} \times 4 \text{ ft}\right) + \\
\left(\frac{1}{2} \times 2 \text{ ft} \times 4 \text{ ft}\right) + \left(\frac{1}{2} \times 5 \text{ ft} \times 4 \text{ ft}\right) + \left(\frac{1}{2} \times 5 \text{ ft} \times 4 \text{ ft}\right)
\]

\[
= 10 \text{ ft}^2 + 8 \text{ ft}^2 + 20 \text{ ft}^2 = 38 \text{ ft}^2
\]

3. **Name of Shape: Rectangular Pyramid, but more specifically a Square Pyramid**

Area of Base: 
\[
3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2
\]

Area of Triangles: 
\[
3 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2
\]

Surface Area: 
\[
9 \text{ m}^2 + 12 \text{ m}^2 + 12 \text{ m}^2 + 12 \text{ m}^2 + 12 \text{ m}^2 = 57 \text{ m}^2
\]

*The error in the solution is the area of the triangles. In order to calculate the correct area of the triangles, you must use the correct formula \(A = \frac{1}{2}bh\). Therefore, the area of each triangle would be \(6 \text{ m}^2\) and not \(12 \text{ m}^2\).*

4. **Name of Shape: Rectangular Prism or, more specifically, a Cube**

Area of Faces: 
\[
3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2
\]

Surface Area: 
\[
9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 = 45 \text{ m}^2
\]

*The surface area is incorrect because the student did not find the sum of all 6 faces. The solution shown above only calculates the sum of 5 faces. Therefore, the correct surface area should be \(9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 = 54 \text{ m}^2 and not 45 \text{ m}^2.*
5. Sofia and Ella are both writing expressions to calculate the surface area of a rectangular prism. However, they wrote different expressions.

a. Examine the expressions below, and determine if they represent the same value. Explain why or why not.

Sofia’s Expression:
\[(3 \text{ cm} \times 4 \text{ cm}) + (3 \text{ cm} \times 4 \text{ cm}) + (3 \text{ cm} \times 5 \text{ cm}) + (3 \text{ cm} \times 5 \text{ cm}) + (4 \text{ cm} \times 5 \text{ cm}) + (4 \text{ cm} \times 5 \text{ cm})\]

Ella’s Expression:
\[2(3 \text{ cm} \times 4 \text{ cm}) + 2(3 \text{ cm} \times 5 \text{ cm}) + 2(4 \text{ cm} \times 5 \text{ cm})\]

Sofia and Ella’s expressions are the same, but Ella used the distributive property to make her expression more compact than Sofia’s.

b. What fact about the surface area of a rectangular prism does Ella’s expression show that Sofia’s does not?

A rectangular prism is composed of three pairs of sides with identical areas.
Lesson 17: From Nets to Surface Area

Classwork

Opening Exercise

a. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.

i. 

ii. How would you write an equation that shows the area of a triangle with base $b$ and height $h$?

b. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.

i. 

ii. How would you write an equation that shows the area of a rectangle with base $b$ and height $h$?
Example 1

Use the net to calculate the surface area of the figure.

Example 2

Use the net to write an expression for surface area.
Exercises

Name the solid the net would create, and then write an expression for the surface area. Use the expression to determine the surface area. Assume that each box on the grid paper represents a 1 cm × 1 cm square. Explain how the expression represents the figure.

1.

2.
3.

4.
Problem Set

Name the shape, and write an expression for surface area. Calculate the surface area of the figure. Assume each box on the grid paper represents a 1 ft. × 1 ft. square.

1. 

2. 

Explain the error in each problem below. Assume each box on the grid paper represents a 1 m × 1 m square.

3. 

Name of Shape: Rectangular Pyramid, but more specifically a Square Pyramid
Area of Base: 3 m × 3 m = 9 m²
Area of Triangles: 3 m × 4 m = 12 m²
Surface Area: 9 m² + 12 m² + 12 m² + 12 m² + 12 m² = 57 m²
4. Name of Shape: Rectangular Prism or, more specifically, a Cube
Area of Faces: $3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2$
Surface Area: $9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 + 9 \text{ m}^2 = 45 \text{ m}^2$

5. Sofia and Ella are both writing expressions to calculate the surface area of a rectangular prism. However, they wrote different expressions.
   a. Examine the expressions below, and determine if they represent the same value. Explain why or why not.

   Sofia’s Expression:
   $$(3 \text{ cm} \times 4 \text{ cm}) + (3 \text{ cm} \times 4 \text{ cm}) + (3 \text{ cm} \times 5 \text{ cm}) + (3 \text{ cm} \times 5 \text{ cm}) + (4 \text{ cm} \times 5 \text{ cm}) + (4 \text{ cm} \times 5 \text{ cm})$$

   Ella’s Expression:
   $$2(3 \text{ cm} \times 4 \text{ cm}) + 2(3 \text{ cm} \times 5 \text{ cm}) + 2(4 \text{ cm} \times 5 \text{ cm})$$

   b. What fact about the surface area of a rectangular prism does Ella’s expression show that Sofia’s does not?
Lesson 18: Determining Surface Area of Three-Dimensional Figures

Student Outcomes

- Students determine that a right rectangular prism has six faces: top and bottom, front and back, and two sides. They determine that surface area is obtained by adding the areas of all the faces and develop the formula $SA = 2lw + 2lh + 2wh$.
- Students develop and apply the formula for the surface area of a cube as $SA = 6s^2$.

Lesson Notes

In order to complete this lesson, each student will need a ruler and the shape template that is attached to the lesson. To save time, teachers should have the shape template cut out for students.

Classwork

Opening Exercise (5 minutes)

In order to complete the Opening Exercise, each student needs a copy of the shape template that is already cut out.

Opening Exercise

a. What three-dimensional figure will the net create?
   
   Rectangular Prism

b. Measure (in inches) and label each side of the figure.
c. Calculate the area of each face, and record this value inside the corresponding rectangle.

![Diagram of a rectangular prism with dimensions and areas labeled.]

d. How did we compute the surface area of solid figures in previous lessons?
   
   *To determine surface area, we found the area of each of the faces and then added those areas.*

e. Write an expression to show how we can calculate the surface area of the figure above.

\[
(4\text{ in.} \times 1\text{ in.}) + (4\text{ in.} \times 2\text{ in.}) + (4\text{ in.} \times 1\text{ in.}) + (4\text{ in.} \times 2\text{ in.}) + (2\text{ in.} \times 1\text{ in.}) + (2\text{ in.} \times 1\text{ in.}) \\
2(4\text{ in.} \times 1\text{ in.}) + 2(4\text{ in.} \times 2\text{ in.}) + 2(2\text{ in.} \times 1\text{ in.})
\]

f. What does each part of the expression represent?

*Each part of the expression represents an area of one face of the given figure. We were able to write a more compacted form because there are three pairs of two faces that are identical.*

g. What is the surface area of the figure?

\[
(4\text{ in.} \times 1\text{ in.}) + (4\text{ in.} \times 2\text{ in.}) + (4\text{ in.} \times 1\text{ in.}) + (4\text{ in.} \times 2\text{ in.}) + (2\text{ in.} \times 1\text{ in.}) + (2\text{ in.} \times 1\text{ in.}) \\
2(4\text{ in.} \times 1\text{ in.}) + 2(4\text{ in.} \times 2\text{ in.}) + 2(2\text{ in.} \times 1\text{ in.}) \\
28\text{ in}^2
\]

**Example 1 (8 minutes)**

- Fold the net used in the Opening Exercise to make a rectangular prism. Have the two faces with the largest area be the bases of the prism.
- Fill in the second row of the table below.

<table>
<thead>
<tr>
<th>Area of Top (base)</th>
<th>Area of Bottom (base)</th>
<th>Area of Front</th>
<th>Area of Back</th>
<th>Area of Left Side</th>
<th>Area of Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 \text{ in}^2</td>
<td>8 \text{ in}^2</td>
<td>4 \text{ in}^2</td>
<td>4 \text{ in}^2</td>
<td>2 \text{ in}^2</td>
<td>2 \text{ in}^2</td>
</tr>
</tbody>
</table>
Lesson 18

- What do you notice about the areas of the faces?
  - Pairs of faces have equal areas.

- What is the relationship between the faces having equal area?
  - The faces that have the same area are across from each other. The bottom and top have the same area, the front and the back have the same area, and the two sides have the same area.

- How do we calculate the area of the two bases of the prism?
  - length × width

- How do we calculate the area of the front and back faces of the prism?
  - length × height

- How do we calculate the area of the right and left faces of the prism?
  - width × height

- Using the name of the dimensions, fill in the third row of the table.

<table>
<thead>
<tr>
<th>Area of Top (base)</th>
<th>Area of Bottom (base)</th>
<th>Area of Front</th>
<th>Area of Back</th>
<th>Area of Left Side</th>
<th>Area of Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 in. × 2 in.</td>
<td>4 in. × 2 in.</td>
<td>2 in. × 2 in.</td>
<td>2 in. × 2 in.</td>
<td>1 in. × 2 in.</td>
<td>1 in. × 2 in.</td>
</tr>
<tr>
<td>8 in²</td>
<td>8 in²</td>
<td>4 in²</td>
<td>4 in²</td>
<td>2 in²</td>
<td>2 in²</td>
</tr>
<tr>
<td>l × w</td>
<td>l × w</td>
<td>l × h</td>
<td>l × h</td>
<td>w × h</td>
<td>w × h</td>
</tr>
</tbody>
</table>

- Examine the rectangular prism below. Complete the table.

<table>
<thead>
<tr>
<th>Area of Top (base)</th>
<th>Area of Bottom (base)</th>
<th>Area of Front</th>
<th>Area of Back</th>
<th>Area of Left Side</th>
<th>Area of Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 cm × 6 cm</td>
<td>15 cm × 6 cm</td>
<td>15 cm × 8 cm</td>
<td>15 cm × 8 cm</td>
<td>8 cm × 6 cm</td>
<td>8 cm × 6 cm</td>
</tr>
<tr>
<td>90 cm²</td>
<td>90 cm²</td>
<td>120 cm²</td>
<td>120 cm²</td>
<td>48 cm²</td>
<td>48 cm²</td>
</tr>
<tr>
<td>l × w</td>
<td>l × w</td>
<td>l × h</td>
<td>l × h</td>
<td>w × h</td>
<td>w × h</td>
</tr>
</tbody>
</table>

- When comparing the methods to finding surface area of the two rectangular prisms, can you develop a general formula?
  - \( SA = l \times w + l \times w + l \times h + l \times h + w \times h + w \times h \)

- Since we use the same expression to calculate the area of pairs of faces, we can use the distributive property to write an equivalent expression for the surface area of the figure that uses half as many terms.

Scaffolding:
Students may benefit from a poster or handout highlighting the length, width, and height of a three-dimensional figure. This poster may also include that \( l = \) length, \( w = \) width, and \( h = \) height.
We have determined that there are two $l \times w$ dimensions. Let’s record that as $2(l \times w)$, or simply $2(l \times w)$. How can we use this knowledge to alter other parts of the formula?

- We also have two $l \times h$, so we can write that as $2(l \times h)$, and we can write the two $w \times h$ as $2(w \times h)$.

Writing each pair in a simpler way, what is the formula to calculate the surface area of a rectangular prism?

- $SA = 2(l \times w) + 2(l \times h) + 2(w \times h)$

Knowing the formula to calculate surface area makes it possible to calculate the surface area without a net.

**Example 2 (5 minutes)**

Work with students to calculate the surface area of the given rectangular prism.

![Example 2](image)

- What are the dimensions of the rectangular prism?
  - *The length is 20 cm, the width is 5 cm, and the height is 9 cm.*

- We will use substitution in order to calculate the area. Substitute the given dimensions into the surface area formula.
  - $SA = 2(20 \text{ cm})(5 \text{ cm}) + 2(20 \text{ cm})(9 \text{ cm}) + 2(5 \text{ cm})(9 \text{ cm})$

- Solve the equation. Remember to use order of operations.
  - $SA = 200 \text{ cm}^2 + 360 \text{ cm}^2 + 90 \text{ cm}^2$
  - $SA = 650 \text{ cm}^2$

**Exercises 1–3 (17 minutes)**

Students work individually to answer the following questions.

**Exercises 1–3**

1. Calculate the surface area of each of the rectangular prisms below.
   a. $3 \text{ in.} \times 12 \text{ in.} \times 2 \text{ in.}$

   - $SA = 2(12 \text{ in.})(2 \text{ in.}) + 2(12 \text{ in.})(3 \text{ in.}) + 2(2 \text{ in.})(3 \text{ in.})$
   - $SA = 48 \text{ in}^2 + 72 \text{ in}^2 + 12 \text{ in}^2$
   - $SA = 132 \text{ in}^2$
b. $SA = 2(8 \text{ m})(6 \text{ m}) + 2(8 \text{ m})(22 \text{ m}) + 2(6 \text{ m})(22 \text{ m})$
   $SA = 96 \text{ m}^2 + 352 \text{ m}^2 + 264 \text{ m}^2$
   $SA = 712 \text{ m}^2$

c. $SA = 2(29 \text{ ft.})(16 \text{ ft.}) + 2(29 \text{ ft.})(23 \text{ ft.}) + 2(16 \text{ ft.})(23 \text{ ft.})$
   $SA = 928 \text{ ft}^2 + 1334 \text{ ft}^2 + 736 \text{ ft}^2$
   $SA = 2998 \text{ ft}^2$

d. $SA = 2(4 \text{ cm})(1.2 \text{ cm}) + 2(4 \text{ cm})(2.8 \text{ cm}) + 2(1.2 \text{ cm})(2.8 \text{ cm})$
   $SA = 9.6 \text{ cm}^2 + 22.4 \text{ cm}^2 + 6.72 \text{ cm}^2$
   $SA = 38.72 \text{ cm}^2$

2. Calculate the surface area of the cube.

   $SA = 2(5 \text{ km})(5 \text{ km}) + 2(5 \text{ km})(5 \text{ km}) + 2(5 \text{ km})(5 \text{ km})$
   $SA = 50 \text{ km}^2 + 50 \text{ km}^2 + 50 \text{ km}^2$
   $SA = 150 \text{ km}^2$
Lesson 18: Determining Surface Area of Three-Dimensional Figures

3. All the edges of a cube have the same length. Tony claims that the formula \( SA = 6s^2 \), where \( s \) is the length of each side of the cube, can be used to calculate the surface area of a cube.
   a. Use the dimensions from the cube in Problem 2 to determine if Tony’s formula is correct.
      
      Tony’s formula is correct because \( SA = 6(5 \text{ km})^2 = 150 \text{ km}^2 \), which is the same surface area when we use the surface area formula for rectangular prisms.

   b. Why does this formula work for cubes?
      
      Each face is a square, and to find the area of a square, you multiply the side lengths together. However, since the side lengths are the same, you can just square the side length. Also, a cube has 6 identical faces, so after calculating the area of one face, we can just multiply this area by 6 to determine the total surface area of the cube.

   c. Becca does not want to try to remember two formulas for surface area, so she is only going to remember the formula for a cube. Is this a good idea? Why or why not?
      
      Becca’s idea is not a good idea. The surface area formula for cubes will only work for cubes because rectangular prisms do not have 6 identical faces. Therefore, Becca also needs to know the surface area formula for rectangular prisms.

Closing (5 minutes)

- Use two different ways to calculate the surface area of a cube with side lengths of 8 cm.
  - \( SA = 2(8 \text{ cm} \times 8 \text{ cm}) + 2(8 \text{ cm} \times 8 \text{ cm}) + 2(8 \text{ cm} \times 8 \text{ cm}) \)
    \( SA = 128 \text{ cm}^2 + 128 \text{ cm}^2 + 128 \text{ cm}^2 \)
    \( SA = 384 \text{ cm}^2 \)
  - \( SA = 6s^2 \)
    \( SA = 6(8 \text{ cm})^2 \)
    \( SA = 384 \text{ cm}^2 \)

- If you had to calculate the surface area of 20 different sized-cubes, which method would you prefer to use, and why?
  - Answers may vary, but most likely students will chose the formula for surface area of a cube because it is a shorter formula, so it would take less time.

Lesson Summary

Surface Area Formula for a Rectangular Prism: \( SA = 2lw + 2lh + 2wh \)

Surface Area Formula for a Cube: \( SA = 6s^2 \)

Exit Ticket (5 minutes)
Lesson 18: Determining Surface Area of Three-Dimensional Figures

Exit Ticket

Calculate the surface area of each figure below. Figures are not drawn to scale.

1. 
   ![Diagram of a rectangular prism with dimensions 10 ft, 12 ft, and 2 ft.]

2. 
   ![Diagram of a cube with dimensions 8 cm.]

Name ___________________________ Date ________________
Exit Ticket Sample Solutions

Calculate the surface area of each figure below. Figures are not drawn to scale.

1. \( SA = 2lw + 2lh + 2wh \)
   \( SA = 2(12 \text{ ft.})(2 \text{ ft.}) + 2(12 \text{ ft.})(10 \text{ ft.}) + 2(2 \text{ ft.})(10 \text{ ft.}) \)
   \( SA = 48 \text{ ft}^2 + 240 \text{ ft}^2 + 40 \text{ ft}^2 \)
   \( SA = 328 \text{ ft}^2 \)

2. \( SA = 6s^2 \)
   \( SA = 6(8 \text{ cm})^2 \)
   \( SA = 6(64 \text{ cm}^2) \)
   \( SA = 384 \text{ cm}^2 \)

Problem Set Sample Solutions

Calculate the surface area of each figure below. Figures are not drawn to scale.

1. \( SA = 2(lh)(7 \text{ in.}) + 2(15 \text{ in.})(7 \text{ in.}) + 2(15 \text{ in.})(15 \text{ in.}) \)
   \( SA = 450 \text{ in}^2 + 210 \text{ in}^2 + 210 \text{ in}^2 \)
   \( SA = 870 \text{ in}^2 \)

2. \( SA = 2(18.7 \text{ cm})(2.3 \text{ cm}) + 2(18.7 \text{ cm})(8.4 \text{ cm}) + 2(2.3 \text{ cm})(8.4 \text{ cm}) \)
   \( SA = 86.02 \text{ cm}^2 + 314.16 \text{ cm}^2 + 38.64 \text{ cm}^2 \)
   \( SA = 438.82 \text{ cm}^2 \)
Lesson 18: Determining Surface Area of Three-Dimensional Figures

3. \( S = 2 \left( \frac{1}{3} \text{ ft.} \right)^2 \)

\[ S = 6 \left( \frac{7}{3} \text{ ft}^2 \right) \]

\[ S = 6 \left( \frac{49}{9} \text{ ft}^2 \right) \]

\[ S = \frac{294}{9} = 32 \frac{2}{3} \text{ ft}^2 \]

4. \( S = 2(32.3 \text{ m})(24.7 \text{ m}) + 2(32.3 \text{ m})(7.9 \text{ m}) + 2(24.7 \text{ m})(7.9 \text{ m}) \]

\[ S = 1595.62 \text{ m}^2 + 510.34 \text{ m}^2 + 390.26 \text{ m}^2 \]

\[ S = 2496.22 \text{ m}^2 \]

5. Write a numerical expression to show how to calculate the surface area of the rectangular prism. Explain each part of the expression.

\[ 2(12 \text{ ft, } \times 3 \text{ ft, }) + 2(12 \text{ ft, } \times 7 \text{ ft, }) + 2(7 \text{ ft, } \times 3 \text{ ft, }) \]

The first part of the expression shows the area of the top and bottom of the rectangular prism. The second part of the expression shows the area of the front and back of the rectangular prism. The third part of the expression shows the area of the two sides of the rectangular prism.

The surface area of the figure is 282 ft^2.

6. When Louie was calculating the surface area for Problem 4, he identified the following:

length = 24.7 ft, width = 32.3 m, and height = 7.9 m.

However, when Rocko was calculating the surface area for the same problem, he identified the following:

length = 32.3 m, width = 24.7 m, and height = 7.9 m.

Would Louie and Rocko get the same answer? Why or why not?

Louie and Rocko would get the same answer because they are still finding the correct area of all six faces of the rectangular prism.
7. Examine the figure below.

![Diagram of a cube with side length 7 m]

a. What is the most specific name of the three-dimensional shape?
   
   *Cube*

b. Write two different expressions for the surface area.

   \[ (7 \text{ m} \times 7 \text{ m}) + (7 \text{ m} \times 7 \text{ m}) + (7 \text{ m} \times 7 \text{ m}) + (7 \text{ m} \times 7 \text{ m}) + (7 \text{ m} \times 7 \text{ m}) + (7 \text{ m} \times 7 \text{ m}) \]

   \[ 6 \times (7 \text{ m})^2 \]

c. Explain how these two expressions are equivalent.

   The two expressions are equivalent because the first expression shows \(7 \text{ m} \times 7 \text{ m}\), which is equivalent to \((7 \text{ m})^2\). Also, the 6 represents the number of times the product \(7 \text{ m} \times 7 \text{ m}\) is added together.
Lesson 18: Determining Surface Area of Three-Dimensional Figures
Lesson 18: Determining Surface Area of Three-Dimensional Figures

Classwork

Opening Exercise

a. What three-dimensional figure will the net create?

b. Measure (in inches) and label each side of the figure.

c. Calculate the area of each face, and record this value inside the corresponding rectangle.

d. How did we compute the surface area of solid figures in previous lessons?

e. Write an expression to show how we can calculate the surface area of the figure above.

f. What does each part of the expression represent?

g. What is the surface area of the figure?
Example 1

Fold the net used in the Opening Exercise to make a rectangular prism. Have the two faces with the largest area be the bases of the prism. Fill in the second row of the table below.

<table>
<thead>
<tr>
<th>Area of Top (base)</th>
<th>Area of Bottom (base)</th>
<th>Area of Front</th>
<th>Area of Back</th>
<th>Area of Left Side</th>
<th>Area of Right Side</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

Examine the rectangular prism below. Complete the table.

![Rectangular Prism Diagram]

<table>
<thead>
<tr>
<th>Area of Top (base)</th>
<th>Area of Bottom (base)</th>
<th>Area of Front</th>
<th>Area of Back</th>
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</tbody>
</table>

Example 2

Exercises 1–3

1. Calculate the surface area of each of the rectangular prisms below.
   a. 
      
      \[
      \text{3 in.} \quad \text{12 in.} \quad \text{2 in.}
      \]
   b. 
      
      \[
      \text{8 m} \quad \text{22 m} \quad \text{6 m}
      \]
   c. 
      
      \[
      \text{23 ft.} \quad \text{29 ft.} \quad \text{16 ft.}
      \]
2. Calculate the surface area of the cube.

3. All the edges of a cube have the same length. Tony claims that the formula $SA = 6s^2$, where $s$ is the length of each side of the cube, can be used to calculate the surface area of a cube.
   
a. Use the dimensions from the cube in Problem 2 to determine if Tony’s formula is correct.

b. Why does this formula work for cubes?

c. Becca does not want to try to remember two formulas for surface area, so she is only going to remember the formula for a cube. Is this a good idea? Why or why not?
Lesson Summary

Surface Area Formula for a Rectangular Prism: \( SA = 2lw + 2lh + 2wh \)
Surface Area Formula for a Cube: \( SA = 6s^2 \)

Problem Set

Calculate the surface area of each figure below. Figures are not drawn to scale.

1. \[ \text{15 in.} \quad \text{7 in.} \quad \text{15 in.} \]
2. \[ \text{8.4 cm} \quad \text{2.3 cm} \quad \text{18.7 cm} \]
3. \[ \text{2 \frac{1}{3} ft.} \quad \text{2 \frac{1}{3} ft.} \quad \text{2 \frac{1}{3} ft.} \]
4. \[ \text{7.9 m} \quad \text{32.3 m} \quad \text{24.7 m} \]
5. Write a numerical expression to show how to calculate the surface area of the rectangular prism. Explain each part of the expression.

![Rectangular Prism Diagram]

6. When Louie was calculating the surface area for Problem 4, he identified the following:
   length = 24.7 m, width = 32.3 m, and height = 7.9 m.

However, when Rocko was calculating the surface area for the same problem, he identified the following:
   length = 32.3 m, width = 24.7 m, and height = 7.9 m.

Would Louie and Rocko get the same answer? Why or why not?

7. Examine the figure below.

![Cube Diagram]

   a. What is the most specific name of the three-dimensional shape?
   b. Write two different expressions for the surface area.
   c. Explain how these two expressions are equivalent.
Lesson 19: Surface Area and Volume in the Real World

Student Outcomes
- Students determine the surface area of three-dimensional figures in real-world contexts.
- Students choose appropriate formulas to solve real-life volume and surface area problems.

Classwork

Fluency Exercise (5 minutes): Area of Shapes

RWBE: Refer to the Rapid White Board Exchange section in the Module Overview for directions to administer an RWBE.

Opening Exercise (4 minutes)

A box needs to be painted. How many square inches will need to be painted to cover every surface?

\[
SA = 2(15 \text{ in.})(12 \text{ in.}) + 2(15 \text{ in.})(6 \text{ in.}) + 2(12 \text{ in.})(6 \text{ in.}) \\
SA = 360 \text{ in}^2 + 180 \text{ in}^2 + 144 \text{ in}^2 \\
SA = 684 \text{ in}^2
\]

A juice box is 4 in. tall, 1 in. wide, and 2 in. long. How much juice fits inside the juice box?

\[
V = 1 \text{ in.} \times 2 \text{ in.} \times 4 \text{ in.} = 8 \text{ in}^3
\]

How did you decide how to solve each problem?

I chose to use surface area to solve the first problem because you would need to know how much area the paint would need to cover. I chose to use volume to solve the second problem because you would need to know how much space is inside the juice box to determine how much juice it can hold.

If students struggle deciding whether to calculate volume or surface area, use the Venn diagram below to help them make the correct decision.
Discussion (5 minutes)

Students need to be able to recognize the difference between volume and surface area. As a class, complete the Venn diagram below so students have a reference when completing the application problems.

Discussion

Volume
- Measures space inside
- Includes only space needed to fill inside
- Is measured in cubic units

Surface Area
- Measures outside surface
- Includes all faces
- Is measured in square units
- Can be measured using a net

Example 1 (5 minutes)

Work through the word problem below with students. Students should be leading the discussion in order for them to be prepared to complete the exercises.

Example 1

Vincent put logs in the shape of a rectangular prism. He built this rectangular prism of logs outside his house. However, it is supposed to snow, and Vincent wants to buy a cover so the logs will stay dry. If the pile of logs creates a rectangular prism with these measurements:

- 33 cm long,
- 12 cm wide,
- 48 cm high,

what is the minimum amount of material needed to make a cover for the wood pile?

- Where do we start?
  - We need to find the size of the cover for the logs, so we need to calculate the surface area. In order to find the surface area, we need to know the dimensions of the pile of logs.

- Why do we need to find the surface area and not the volume?
  - We want to know the size of the cover Vincent wants to buy. If we calculated volume, we would not have the information Vincent needs when he goes shopping for a cover.

- What are the dimensions of the pile of logs?
  - The length is 33 cm, the width is 12 cm, and the height is 48 cm.

Scaffolding:

- Add to the poster or handout made in the previous lesson showing that long represents length, wide represents width, and high represents height.
- Later, students will have to recognize that deep also represents height. Therefore, this vocabulary word should also be added to the poster.
How do we calculate the surface area to determine the size of the cover?

- We can use the surface area formula for a rectangular prism.

\[
SA = 2(33 \text{ cm})(12 \text{ cm}) + 2(33 \text{ cm})(48 \text{ cm}) + 2(12 \text{ cm})(48 \text{ cm})
\]

\[
SA = 792 \text{ cm}^2 + 3168 \text{ cm}^2 + 1152 \text{ cm}^2
\]

\[
SA = 5112 \text{ cm}^2
\]

What is different about this problem from other surface area problems of rectangular prisms you have encountered? How does this change the answer?

- If Vincent just wants to cover the wood to keep it dry, he does not need to cover the bottom of the pile of logs. Therefore, the cover can be smaller.

How can we change our answer to find the exact size of the cover Vincent needs?

- We know the area of the bottom of the pile of firewood has the dimensions 33 cm and 12 cm. We can calculate the area and subtract this area from the total surface area.

\[
\text{The area of the bottom of the pile of firewood is } 396 \text{ cm}^2; \text{ therefore, the total surface area of the cover would need to be } 5112 \text{ cm}^2 - 396 \text{ cm}^2 = 4716 \text{ cm}^2.
\]

Exercises 1–6 (17 minutes)

Students complete the volume and surface area problems in small groups.

**Exercises 1–6**

Use your knowledge of volume and surface area to answer each problem.

1. Quincy Place wants to add a pool to the neighborhood. When determining the budget, Quincy Place determined that it would also be able to install a baby pool that requires less than \(111\) cubic feet of water. Quincy Place has three different models of a baby pool to choose from.

   - **Choice One:** \(5 \text{ feet} \times 5 \text{ feet} \times 1 \text{ foot}
   - **Choice Two:** \(4 \text{ feet} \times 3 \text{ feet} \times 1 \text{ foot}
   - **Choice Three:** \(4 \text{ feet} \times 2 \text{ feet} \times 2 \text{ feet}

Which of these choices is best for the baby pool? Why are the others not good choices?

**Choice One Volume:** \(5 \text{ ft} \times 5 \text{ ft} \times 1 \text{ ft} = 25 \text{ cubic feet}

**Choice Two Volume:** \(4 \text{ ft} \times 3 \text{ ft} \times 1 \text{ ft} = 12 \text{ cubic feet}

**Choice Three Volume:** \(4 \text{ ft} \times 2 \text{ ft} \times 2 \text{ ft} = 16 \text{ cubic feet}

**Choice Two is within the budget because it holds less than 15 cubic feet of water.** The other two choices do not work because they require too much water, and Quincy Place will not be able to afford the amount of water it takes to fill the baby pool.

2. A packaging firm has been hired to create a box for baby blocks. The firm was hired because it could save money by creating a box using the least amount of material. The packaging firm knows that the volume of the box must be \(18 \text{ cm}^3\).

   a. What are possible dimensions for the box if the volume must be exactly \(18 \text{ cm}^3\)?

   - **Choice 1:** \(1 \text{ cm} \times 1 \text{ cm} \times 18 \text{ cm}
   - **Choice 2:** \(1 \text{ cm} \times 2 \text{ cm} \times 9 \text{ cm}
   - **Choice 3:** \(1 \text{ cm} \times 3 \text{ cm} \times 6 \text{ cm}
   - **Choice 4:** \(2 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}

   **Choice Two is within the budget because it holds less than 15 cubic feet of water.**
b. Which set of dimensions should the packaging firm choose in order to use the least amount of material? Explain.

*Choice 1:* \( SA = 2(1 \text{ cm})(1 \text{ cm}) + 2(1 \text{ cm})(18 \text{ cm}) + 2(1 \text{ cm})(18 \text{ cm}) = 74 \text{ cm}^2 \)

*Choice 2:* \( SA = 2(1 \text{ cm})(2 \text{ cm}) + 2(1 \text{ cm})(9 \text{ cm}) + 2(2 \text{ cm})(9 \text{ cm}) = 58 \text{ cm}^2 \)

*Choice 3:* \( SA = 2(1 \text{ cm})(3 \text{ cm}) + 2(1 \text{ cm})(6 \text{ cm}) + 2(3 \text{ cm})(6 \text{ cm}) = 54 \text{ cm}^2 \)

*Choice 4:* \( SA = 2(2 \text{ cm})(3 \text{ cm}) + 2(2 \text{ cm})(3 \text{ cm}) + 2(3 \text{ cm})(3 \text{ cm}) = 42 \text{ cm}^2 \)

The packaging firm should choose Choice 4 because it requires the least amount of material. In order to find the amount of material needed to create a box, the packaging firm would have to calculate the surface area of each box. The box with the smallest surface area requires the least amount of material.

3. A gift has the dimensions of 50 cm \( \times \) 35 cm \( \times \) 5 cm. You have wrapping paper with dimensions of 75 cm \( \times \) 60 cm. Do you have enough wrapping paper to wrap the gift? Why or why not?

*Surface Area of the Present:* \( SA = 2(50 \text{ cm})(35 \text{ cm}) + 2(50 \text{ cm})(5 \text{ cm}) + 2(35 \text{ cm})(5 \text{ cm}) = 3500 \text{ cm}^2 + 500 \text{ cm}^2 + 350 \text{ cm}^2 = 4350 \text{ cm}^2 \)

*Area of Wrapping Paper:* \( A = 75 \text{ cm} \times 60 \text{ cm} = 4200 \text{ cm}^2 \)

I do have enough paper to wrap the present because the present requires 4,350 square centimeters of paper, and I have 4,500 square centimeters of wrapping paper.

4. Tony bought a flat rate box from the post office to send a gift to his mother for Mother’s Day. The dimensions of the medium size box are 14 inches \( \times \) 12 inches \( \times \) 3.5 inches. What is the volume of the largest gift he can send to his mother?

*Volume of the Box:* \( 14 \text{ in.} \times 12 \text{ in.} \times 3.5 \text{ in.} = 588 \text{ in}^3 \)

Tony would have 588 cubic inches of space to fill with a gift for his mother.

5. A cereal company wants to change the shape of its cereal box in order to attract the attention of shoppers. The original cereal box has dimensions of 8 inches \( \times \) 3 inches \( \times \) 11 inches. The new box the cereal company is thinking of would have dimensions of 10 inches \( \times \) 10 inches \( \times \) 3 inches.

a. Which box holds more cereal?

*Volume of Original Box:* \( V = 8 \text{ in.} \times 3 \text{ in.} \times 11 \text{ in.} = 264 \text{ in}^3 \)

*Volume of New Box:* \( V = 10 \text{ in.} \times 10 \text{ in.} \times 3 \text{ in.} = 300 \text{ in}^3 \)

The new box holds more cereal because it has a larger volume.

b. Which box requires more material to make?

*Surface Area of Original Box:* \( SA = 2(8 \text{ in.})(3 \text{ in.}) + 2(8 \text{ in.})(11 \text{ in.}) + 2(3 \text{ in.})(11 \text{ in.}) = 48 \text{ in}^2 + 176 \text{ in}^2 + 66 \text{ in}^2 = 290 \text{ in}^2 \)

*Surface Area of New Box:* \( SA = 2(10 \text{ in.})(10 \text{ in.}) + 2(10 \text{ in.})(3 \text{ in.}) + 2(10 \text{ in.})(3 \text{ in.}) = 200 \text{ in}^2 + 60 \text{ in}^2 + 60 \text{ in}^2 = 320 \text{ in}^2 \)

The new box requires more material than the original box because the new box has a larger surface area.
6. Cinema theaters created a new popcorn box in the shape of a rectangular prism. The new popcorn box has a length of 6 inches, a width of 3.5 inches, and a height of 3.5 inches but does not include a lid.

![Image of a rectangular prism]

a. How much material is needed to create the box?

Surface Area of the Box: \[ SA = 2(6 \text{ in.})(3.5 \text{ in.}) + 2(6 \text{ in.})(3.5 \text{ in.}) + 2(3.5 \text{ in.})(3.5 \text{ in.}) = 42 \text{ in}^2 + 42 \text{ in}^2 + 24.5 \text{ in}^2 = 108.5 \text{ in}^2 \]

The box does not have a lid, so we have to subtract the area of the lid from the surface area.

Area of Lid: \[ 6 \text{ in.} \times 3.5 \text{ in.} = 21 \text{ in}^2 \]

Total Surface Area: \[ 108.5 \text{ in}^2 - 21 \text{ in}^2 = 87.5 \text{ in}^2 \]

87.5 square inches of material is needed to create the new popcorn box.

b. How much popcorn does the box hold?

Volume of the Box: \[ V = 6 \text{ in.} \times 3.5 \text{ in.} \times 3.5 \text{ in.} = 73.5 \text{ in}^3 \]

Closing (4 minutes)

- Is it possible for two containers having the same volume to have different surface areas? Explain.
  - Yes, it is possible to have two containers to have the same volume but different surface areas. This was the case in Exercise 2. All four boxes would hold the same amount of baby blocks (same volume), but required a different amount of material (surface area) to create the box.

- If you want to create an open box with dimensions 3 inches × 4 inches × 5 inches, which face should be the base if you want to minimize the amount of material you use?
  - The face with dimensions 4 inches × 5 inches should be the base because that face would have the largest area.

If students have a hard time understanding an open box, use a shoe box to demonstrate the difference between a closed box and an open box.

Exit Ticket (5 minutes)
Lesson 19: Surface Area and Volume in the Real World

Exit Ticket

Solve the word problem below.

Kelly has a rectangular fish aquarium with an open top that measures 18 inches long, 8 inches wide, and 12 inches tall.

a. What is the maximum amount of water in cubic inches the aquarium can hold?

b. If Kelly wanted to put a protective covering on the four glass walls of the aquarium, how big does the cover have to be?
Exit Ticket Sample Solutions

Solve the word problem below.

Kelly has a rectangular fish aquarium that measures 18 inches long, 8 inches wide, and 12 inches tall.

a. What is the maximum amount of water the aquarium can hold?

Volume of the Aquarium: \( V = 18 \text{ in.} \times 8 \text{ in.} \times 12 \text{ in.} = 1728 \text{ in}^3 \)

The maximum amount of water the aquarium can hold is 1,728 cubic inches.

b. If Kelly wanted to put a protective covering on the four glass walls of the aquarium, how big does the cover have to be?

Surface Area of the Aquarium: \( SA = 2(18 \text{ in.})(8 \text{ in.}) + 2(18 \text{ in.})(12 \text{ in.}) + 2(8 \text{ in.})(12 \text{ in.}) = 288 \text{ in}^2 + 432 \text{ in}^2 + 192 \text{ in}^2 = 912 \text{ in}^2 \)

We only need to cover the four glass walls, so we can subtract the area of both the top and bottom of the aquarium.

Area of Top: \( A = 18 \text{ in.} \times 8 \text{ in.} = 144 \text{ in}^2 \)

Area of Bottom: \( A = 18 \text{ in.} \times 8 \text{ in.} = 144 \text{ in}^2 \)

Surface Area of the Four Walls: \( SA = 912 \text{ in}^2 - 144 \text{ in}^2 - 144 \text{ in}^2 = 624 \text{ in}^2 \)

Kelly would need 624 square inches to cover the four walls of the aquarium.

Problem Set Sample Solutions

Solve each problem below.

1. Dante built a wooden, cubic toy box for his son. Each side of the box measures 2 feet.

   a. How many square feet of wood did he use to build the box?

   Surface Area of the Box: \( SA = 6(2 \text{ ft})^2 = 6(4 \text{ ft}^2) = 24 \text{ ft}^2 \)

   Dante would need 24 square feet of wood to build the box.

   b. How many cubic feet of toys will the box hold?

   Volume of the Box: \( V = 2 \text{ ft.} \times 2 \text{ ft.} \times 2 \text{ ft.} = 8 \text{ ft}^3 \)

   The toy box would hold 8 cubic feet of toys.

2. A company that manufactures gift boxes wants to know how many different sized boxes having a volume of 50 cubic centimeters it can make if the dimensions must be whole centimeters.

   a. List all the possible whole number dimensions for the box.

   Choice One: \( 1 \text{ cm} \times 1 \text{ cm} \times 50 \text{ cm} \)

   Choice Two: \( 1 \text{ cm} \times 2 \text{ cm} \times 25 \text{ cm} \)

   Choice Three: \( 1 \text{ cm} \times 5 \text{ cm} \times 10 \text{ cm} \)

   Choice Four: \( 2 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm} \)
b. Which possibility requires the least amount of material to make?

Choice One: \(SA = 2(1\text{ cm})(1\text{ cm}) + 2(1\text{ cm})(50\text{ cm}) + 2(1\text{ cm})(50\text{ cm}) = 2\text{ cm}^2 + 100\text{ cm}^2 + 100\text{ cm}^2 = 202\text{ cm}^2\)

Choice Two: \(SA = 2(1\text{ cm})(2\text{ cm}) + 2(1\text{ cm})(25\text{ cm}) + 2(2\text{ cm})(25\text{ cm}) = 4\text{ cm}^2 + 50\text{ cm}^2 + 100\text{ cm}^2 = 154\text{ cm}^2\)

Choice Three: \(SA = 2(1\text{ cm})(5\text{ cm}) + 2(1\text{ cm})(10\text{ cm}) + 2(5\text{ cm})(10\text{ cm}) = 10\text{ cm}^2 + 20\text{ cm}^2 + 100\text{ cm}^2 = 130\text{ cm}^2\)

Choice Four: \(SA = 2(2\text{ cm})(5\text{ cm}) + 2(2\text{ cm})(5\text{ cm}) + 2(5\text{ cm})(5\text{ cm}) = 20\text{ cm}^2 + 20\text{ cm}^2 + 50\text{ cm}^2 = 90\text{ cm}^2\)

Choice Four requires the least amount of material because it has the smallest surface area.

c. Which box would you recommend the company use? Why?

I would recommend the company use the box with dimensions of 2 cm \(\times\) 5 cm \(\times\) 5 cm (Choice Four) because it requires the least amount of material to make; so, it would cost the company the least amount of money to make.

3. A rectangular box of rice is shown below. How many cubic inches of rice can fit inside?

![Rectangular box diagram]

Volume of the Rice Box: \(V = 15\frac{1}{3}\text{ in.} \times 7\frac{2}{3}\text{ in.} \times 6\frac{1}{3}\text{ in.} = \frac{20102}{27}\text{ in}^3 = 744\frac{14}{27}\text{ in}^3\)

4. The Mars Cereal Company has two different cereal boxes for Mars Cereal. The large box is 8 inches wide, 11 inches high, and 3 inches deep. The small box is 6 inches wide, 10 inches high, and 2.5 inches deep.

a. How much more cardboard is needed to make the large box than the small box?

Surface Area of the Large Box: \(SA = 2(8\text{ in.})(11\text{ in.}) + 2(8\text{ in.})(3\text{ in.}) + 2(11\text{ in.})(3\text{ in.}) = 176\text{ in}^2 + 48\text{ in}^2 + 66\text{ in}^2 = 290\text{ in}^2\)

Surface Area of the Small Box: \(SA = 2(6\text{ in.})(10\text{ in.}) + 2(6\text{ in.})(2.5\text{ in.}) + 2(10\text{ in.})(2.5\text{ in.}) = 120\text{ in}^2 + 30\text{ in}^2 + 50\text{ in}^2 = 200\text{ in}^2\)

Difference: \(290\text{ in}^2 - 200\text{ in}^2 = 90\text{ in}^2\)

The large box requires 90 square inches more material than the small box.

b. How much more cereal does the large box hold than the small box?

Volume of the Large Box: \(V = 8\text{ in.} \times 11\text{ in.} \times 3\text{ in.} = 264\text{ in}^3\)

Volume of the Small Box: \(V = 6\text{ in.} \times 10\text{ in.} \times 2.5\text{ in.} = 150\text{ in}^3\)

Difference: \(264\text{ in}^3 - 150\text{ in}^3 = 114\text{ in}^3\)

The large box holds 114 cubic inches more cereal than the small box.
Lesson 19: Surface Area and Volume in the Real World

5. A swimming pool is 8 meters long, 6 meters wide, and 2 meters deep. The water-resistant paint needed for the pool costs $6 per square meter. How much will it cost to paint the pool?
   a. How many faces of the pool do you have to paint?
      You will have to paint 5 faces.
   b. How much paint (in square meters) do you need to paint the pool?
      \[ SA = 2(8 \text{ m} \times 6 \text{ m}) + 2(8 \text{ m} \times 2 \text{ m}) + 2(6 \text{ m} \times 2 \text{ m}) = 96 \text{ m}^2 + 32 \text{ m}^2 + 24 \text{ m}^2 = 152 \text{ m}^2 \]
      \[ \text{Area of Top of Pool} = 8 \text{ m} \times 6 \text{ m} = 48 \text{ m}^2 \]
      \[ \text{Total Paint Needed} = 152 \text{ m}^2 - 48 \text{ m}^2 = 104 \text{ m}^2 \]
   c. How much will it cost to paint the pool?
      \[ 104 \text{ m}^2 \times 6 \text{ m} = \$624 \]
      It will cost $624 to paint the pool.

6. Sam is in charge of filling a rectangular hole with cement. The hole is 9 feet long, 3 feet wide, and 2 feet deep. How much cement will Sam need?
   \[ V = 9 \text{ ft} \times 3 \text{ ft} \times 2 \text{ ft} = 54 \text{ ft}^3 \]
   Sam will need 54 cubic feet of cement to fill the hole.

7. The volume of Box D subtracted from the volume of Box C is 23.14 cubic centimeters. Box D has a volume of 10.115 cubic centimeters.
   a. Let \( C \) be the volume of Box C in cubic centimeters. Write an equation that could be used to determine the volume of Box C.
      \[ C - 10.115 \text{ cm}^3 = 23.14 \text{ cm}^3 \]
   b. Solve the equation to determine the volume of Box C.
      \[ C = 10.115 \text{ cm}^3 + 10.115 \text{ cm}^3 = 23.14 \text{ cm}^3 + 10.115 \text{ cm}^3 \]
      \[ C = 33.255 \text{ cm}^3 \]
   c. The volume of Box C is one-tenth the volume of another box, Box E. Let \( E \) represent the volume of Box E in cubic centimeters. Write an equation that could be used to determine the volume of Box E, using the result from part (b).
      \[ 33.255 \text{ cm}^3 = \frac{1}{10}E \]
   d. Solve the equation to determine the volume of Box E.
      \[ 33.255 \text{ cm}^3 \times \frac{1}{10} = \frac{1}{10}E \times \frac{1}{10} \]
      \[ 332.55 \text{ cm}^3 = E \]
Area of Shapes

1. 
   \[ A = 80 \text{ ft}^2 \]

2. 
   \[ A = 30 \text{ m}^2 \]

3. 
   \[ A = 484 \text{ in}^2 \]

4. 
   \[ A = 1,029 \text{ cm}^2 \]
Lesson 19: Surface Area and Volume in the Real World

5. \[ A = 72 \text{ ft}^2 \]

6. \[ A = 156 \text{ km}^2 \]

7. \[ A = 110 \text{ in}^2 \]

8. \[ A = 192 \text{ cm}^2 \]
9. \[ A = 576 \text{ m}^2 \]

10. \[ A = 1,476 \text{ ft}^2 \]
Lesson 19: Surface Area and Volume in the Real World

Classwork

Opening Exercise

A box needs to be painted. How many square inches will need to be painted to cover every surface?

A juice box is 4 in. tall, 1 in. wide, and 2 in. long. How much juice fits inside the juice box?

How did you decide how to solve each problem?

Discussion

![Venn Diagram]
Example 1

Vincent put logs in the shape of a rectangular prism. He built this rectangular prism of logs outside his house. However, it is supposed to snow, and Vincent wants to buy a cover so the logs will stay dry. If the pile of logs creates a rectangular prism with these measurements:

33 cm long, 12 cm wide, and 48 cm high,

what is the minimum amount of material needed to make a cover for the wood pile?

Exercises 1–6

Use your knowledge of volume and surface area to answer each problem.

1. Quincy Place wants to add a pool to the neighborhood. When determining the budget, Quincy Place determined that it would also be able to install a baby pool that required less than 15 cubic feet of water. Quincy Place has three different models of a baby pool to choose from.

   Choice One: 5 feet × 5 feet × 1 foot
   Choice Two: 4 feet × 3 feet × 1 foot
   Choice Three: 4 feet × 2 feet × 2 feet

   Which of these choices is best for the baby pool? Why are the others not good choices?
2. A packaging firm has been hired to create a box for baby blocks. The firm was hired because it could save money by creating a box using the least amount of material. The packaging firm knows that the volume of the box must be 18 cm³.
   a. What are possible dimensions for the box if the volume must be exactly 18 cm³?
   b. Which set of dimensions should the packaging firm choose in order to use the least amount of material? Explain.

3. A gift has the dimensions of 50 cm × 35 cm × 5 cm. You have wrapping paper with dimensions of 75 cm × 60 cm. Do you have enough wrapping paper to wrap the gift? Why or why not?

4. Tony bought a flat rate box from the post office to send a gift to his mother for Mother’s Day. The dimensions of the medium size box are 14 inches × 12 inches × 3.5 inches. What is the volume of the largest gift he can send to his mother?
5. A cereal company wants to change the shape of its cereal box in order to attract the attention of shoppers. The original cereal box has dimensions of 8 inches \( \times \) 3 inches \( \times \) 11 inches. The new box the cereal company is thinking of would have dimensions of 10 inches \( \times \) 10 inches \( \times \) 3 inches.

   a. Which box holds more cereal?

   b. Which box requires more material to make?

6. Cinema theaters created a new popcorn box in the shape of a rectangular prism. The new popcorn box has a length of 6 inches, a width of 3.5 inches, and a height of 3.5 inches but does not include a lid.

   a. How much material is needed to create the box?

   b. How much popcorn does the box hold?
Problem Set

Solve each problem below.

1. Dante built a wooden, cubic toy box for his son. Each side of the box measures 2 feet.
   a. How many square feet of wood did he use to build the box?
   b. How many cubic feet of toys will the box hold?

2. A company that manufactures gift boxes wants to know how many different sized boxes having a volume of 50 cubic centimeters it can make if the dimensions must be whole centimeters.
   a. List all the possible whole number dimensions for the box.
   b. Which possibility requires the least amount of material to make?
   c. Which box would you recommend the company use? Why?

3. A rectangular box of rice is shown below. How many cubic inches of rice can fit inside?

4. The Mars Cereal Company has two different cereal boxes for Mars Cereal. The large box is 8 inches wide, 11 inches high, and 3 inches deep. The small box is 6 inches wide, 10 inches high, and 2.5 inches deep.
   a. How much more cardboard is needed to make the large box than the small box?
   b. How much more cereal does the large box hold than the small box?

5. A swimming pool is 8 meters long, 6 meters wide, and 2 meters deep. The water-resistant paint needed for the pool costs $6 per square meter. How much will it cost to paint the pool?
   a. How many faces of the pool do you have to paint?
   b. How much paint (in square meters) do you need to paint the pool?
   c. How much will it cost to paint the pool?

6. Sam is in charge of filling a rectangular hole with cement. The hole is 9 feet long, 3 feet wide, and 2 feet deep. How much cement will Sam need?
7. The volume of Box D subtracted from the volume of Box C is 23.14 cubic centimeters. Box D has a volume of 10.115 cubic centimeters.

a. Let $C$ be the volume of Box C in cubic centimeters. Write an equation that could be used to determine the volume of Box C.

b. Solve the equation to determine the volume of Box C.

c. The volume of Box C is one-tenth the volume another box, Box E. Let $E$ represent the volume of Box E in cubic centimeters. Write an equation that could be used to determine the volume of Box E, using the result from part (b).

d. Solve the equation to determine the volume of Box E.