LESSONS 7, 8, & 9 are the FOURTH set of Eureka Lessons for Unit 1. Also Includes four fluency drills, 2 in lesson 8 and 2 in lesson 9. 😊

Lesson 7
- Pages 2-6 Teacher Pages Addition & Subtraction of Rational Numbers
- Pages 7-9 Exit Ticket w/ solutions for Lesson 7
- Pages 10-14 Student pages for Lesson 7

Lesson 8
- Pages 15-21 Teacher Pages Applying Properties of Operations to Add & Subtract Rational Numbers
- Pages 22-24 Exit Ticket w/ solutions for Lesson 8
- Pages 25-28 Fluency Drills (2 of them)
- Pages 29-33 Student pages for Lesson 8

Lesson 9
- Pages 34-38 Teacher Pages Applying Properties of Operations to Add & Subtract Rational Numbers
- Pages 39-41 Exit Ticket w/ solutions for Lesson 9
- Pages 42-45 Fluency Drills (2 of them)
- Pages 46-48 Student pages for Lesson 9
Lesson 7: Addition and Subtraction of Rational Numbers

Student Outcomes

- Students recognize that the rules for adding and subtracting integers apply to rational numbers.
- Given a number line, students use arrows to model rational numbers where the length of the arrow is the absolute value of the rational number and the sign of the rational number is determined by the direction of the arrow with respect to the number line.
- Students locate the sum \( p + q \) of two rational numbers on a number line by placing the tail of the arrow for \( q \) at \( p \) and locating \( p + q \) at the head of the arrow. They create an arrow for the difference \( p - q \) by first rewriting the difference as a sum, \( p + (-q) \), and then locating the sum.

Classwork

Exercise 1 (5 minutes)

Students answer the following question independently as the teacher circulates around the room providing guidance and feedback as needed. Students focus on how to represent the answer using both an equation and a number line diagram.

Exercise 1

Suppose a 7th grader’s birthday is today, and she is 12 years old. How old was she 3\(\frac{1}{2}\) years ago? Write an equation and use a number line to model your answer.

\[ 12 + (-3\frac{1}{2}) = 8\frac{1}{2} \text{ or } 12 - 3\frac{1}{2} = 8\frac{1}{2} \]

Example 1 (5 minutes): Representing Sums of Rational Numbers on a Number Line

Teacher leads a whole group instruction illustrating the sum of \( 12 + (-3\frac{1}{2}) \) on a number line. Elicit student responses to assist in creating the steps. Students record the steps and diagram.

Example 1: Representing Sums of Rational Numbers on a Number Line

a. Place the tail of the arrow on 12.

b. The length of the arrow is the absolute value of \(-3\frac{1}{2}\) or \(3\frac{1}{2}\).

c. The direction of the arrow is to the left since you are adding a negative number to 12.

Scaffolding:

- Laminate an index card with the steps for Examples 1 and 2 and the number line diagram so that students can easily refer to it.
Lesson 7: Addition and Subtraction of Rational Numbers

Example 2: Representing Differences of Rational Numbers on a Number Line

Teacher leads a whole group instruction illustrating how to find the difference of $1 - 2 \frac{1}{4}$ on a number line. Elicit student responses to assist in creating the steps. Students record the steps and diagram.

**Example 2**: Representing Differences of Rational Numbers on a Number Line

- **a.** Rewrite the difference $1 - 2 \frac{1}{4}$ as a sum: $1 + (-2 \frac{1}{4})$.

Now follow the steps to represent the sum:

- **b.** Place the tail of the arrow on 1.

- **c.** The length of the arrow is the absolute value of $-2 \frac{1}{4}$: $|-2 \frac{1}{4}| = 2 \frac{1}{4}$.

- **d.** The direction of the arrow is to the left since you are adding a negative number to 1.

Draw the number line model in the space below.

$$1 + (-2 \frac{1}{4}) = -1 \frac{1}{4}$$
Exercise 3 (3 minutes)

Exercise 3
Find the following difference, and represent it on a number line. \(-5\frac{1}{2} - (-8)\).

Exercise 4 (10 minutes)
Next, students work independently in Exercise 4 to create a number line model to represent each sum or difference. After 5–7 minutes, students are selected to share their responses and work with the class.

Exercise 4
Find the following sums and differences using a number line model.

a. \(-6 + 5\frac{1}{4}\)
\[-6 + 5\frac{1}{4} = -\frac{3}{4}\]

b. \(7 - (-0.9)\)
\[7 + (0.9) = 7.9\]

c. \(2.5 + (-\frac{1}{2})\)
\[2.5 + (-0.5) = 2\]

d. \(-\frac{1}{4} + 4\)
\[-\frac{1}{4} + 4 = 3\frac{3}{4}\]

e. \(\frac{1}{2} - (-3)\)
\[\frac{1}{2} + 3 = 3\frac{1}{2}\]

Scaffolding:
- Ask students to explain and justify what they drew to check for understanding.
- Ask probing questions such as “Why does your arrow go to the right?”
Exercise 5 (6 minutes)

Exercise 5
Create an equation and number line diagram to model each answer.

a. Samantha owes her father $7. She just got paid $5.50 for babysitting. If she gives that money to her dad, how much will she still owe him?

\[-7 + 5.50 = -1.50, \text{ She still owes him } $1.50.\]

b. At the start of a trip, a car’s gas tank contains 12 gallons of gasoline. During the trip, the car consumes \(10 \frac{1}{8}\) gallons of gasoline. How much gasoline is left in the tank?

\[12 + \left(-10 \frac{1}{8}\right) = 1\frac{7}{8} \text{ or } 12 - 10 \frac{1}{8} = 1\frac{7}{8} \text{ gallons}\]

Follow-Up Discussion
For Exercise 5(a) discuss with students how the mathematical answer of \(-1.50\) means Samantha owes her father $1.50 and that we do not say she owes her father \(-$1.50.\)
Closing (3 minutes)

- What challenges do you face when using the number line model to add non-integer rational numbers?
  - Answers will vary.

- When using a number line to model $8 - (-2.1)$, how many units do we move from 8 and in what direction? Where is the tail of the arrow, and where is the head? What does your arrow represent?
  - First, we would change the expression to an addition expression, $8 + 2.1$. The tail of the arrow would start at 8, the first addend. The arrow would be 2.1 units long and pointing to the right, which would mean the arrow would end on 10.1. The arrow represents the second addend.

Lesson Summary

The rules for adding and subtracting integers apply to all rational numbers.

The sum of two rational numbers (e.g., $-1 + 4.3$) can be found on the number line by placing the tail of an arrow at $-1$ and locating the head of the arrow 4.3 units to the right to arrive at the sum, which is 3.3.

To model the difference of two rational numbers on a number line (e.g., $-5.7 - 3$), first rewrite the difference as a sum, $-5.7 + (-3)$, and then follow the steps for locating a sum. Place a single arrow with its tail at $-5.7$ and the head of the arrow 3 units to the left to arrive at $-8.7$.

Exit Ticket (5 minutes)
Lesson 7: Addition and Subtraction of Rational Numbers

Exit Ticket

At the beginning of the summer, the water level of a pond is 2 feet below its normal level. After an unusually dry summer, the water level of the pond dropped another $1 \frac{1}{3}$ feet.

1. Use a number line diagram to model the pond’s current water level in relation to its normal water level.

2. Write an equation to show how far above or below the normal water level the pond is at the end of the summer.
Exit Ticket Sample Solutions

At the beginning of the summer, the water level of a pond is 2 feet below its normal level. After an unusually dry summer, the water level of the pond dropped another $1 \frac{1}{3}$ feet.

1. Use a number line diagram to model the pond’s current water level in relation to its normal water level.

   Move $1 \frac{1}{3}$ units to the left of $-2$. $-2 - 1 \frac{1}{3} = -3 \frac{1}{3}$

   ![Number line diagram]

2. Write an equation to show how far above or below the normal water level the pond is at the end of the summer.

   $-2 - 1 \frac{1}{3} = -3 \frac{1}{3}$ or $-2 + (-1 \frac{1}{3}) = -3 \frac{1}{3}$

Problem Set Sample Solutions

Represent each of the following problems using both a number line diagram and an equation.

1. A bird that was perched atop a $15 \frac{1}{2}$-foot tree dives down six feet to a branch below. How far above the ground is the bird’s new location?

   $15 \frac{1}{2} + (-6) = 9 \frac{1}{2}$ or $15 \frac{1}{2} - 6 = 9 \frac{1}{2}$

   The bird is $9 \frac{1}{2}$ feet above the ground.

   ![Number line diagram]

2. Mariah had owed her grandfather $2.25 but was recently able to pay him back $1.50. How much does Mariah currently owe her grandfather?

   $-2.25 + 1.50 = -0.75$

   Mariah owes her grandfather 75 cents.

   ![Number line diagram]
3. Jake is hiking a trail that leads to the top of a canyon. The trail is 4.2 miles long, and Jake plans to stop for lunch after he completes 1.6 miles. How far from the top of the canyon will Jake be when he stops for lunch?

\[ 4.2 - 1.6 = 2.6 \]

Jake will be 2.6 miles from the top of the canyon.

4. Sonji and her friend Rachel are competing in a running race. When Sonji is 0.4 miles from the finish line, she notices that her friend Rachel has fallen. If Sonji runs one tenth of a mile back to help her friend, how far will she be from the finish line?

\[ -0.4 + (-0.1) = -0.5 \quad \text{or} \quad -0.4 - 0.1 = -0.5 \]

Sonji will be 0.5 miles from the finish line.

5. Mr. Henderson did not realize his checking account had a balance of $100 when he used his debit card for a $317.25 purchase. What is his checking account balance after the purchase?

\[ 200 + (-317.25) = -117.25 \quad \text{or} \quad 200 - 317.25 = -117.25 \]

Mr. Henderson’s checking account balance will be $-117.25.

6. If the temperature is $-3^\circ F$ at 10:00 p.m. and the temperature falls four degrees overnight, what is the resulting temperature?

\[ -3 - 4 = -3 + (-4) = -7 \]

The resulting temperature is $-7^\circ F$. 
Lesson 7: Addition and Subtraction of Rational Numbers

Classwork

Exercise 1: Real-World Connection to Adding and Subtracting Rational Numbers

Suppose a 7th grader’s birthday is today, and she is 12 years old. How old was she \(3 \frac{1}{2}\) years ago? Write an equation and use a number line to model your answer.

Example 1: Representing Sums of Rational Numbers on a Number Line

a. Place the tail of the arrow on 12.

b. The length of the arrow is the absolute value of \(-3 \frac{1}{2}\), \(|-3 \frac{1}{2}| = 3 \frac{1}{2}\).

c. The direction of the arrow is to the left since you are adding a negative number to 12.

Draw the number line model in the space below.
Exercise 2

Find the following sum using a number line diagram. \(-2\frac{1}{4} + 5\).

Example 2: Representing Differences of Rational Numbers on a Number Line

a. Rewrite the difference \(1 - 2\frac{1}{4}\) as a sum: \(1 + (-2\frac{1}{4})\).

Now follow the steps to represent the sum:

b. Place the tail of the arrow on 1.

c. The length of the arrow is the absolute value of \(-2\frac{1}{4}\); \(|-2\frac{1}{4}| = 2\frac{1}{4}\).

d. The direction of the arrow is to the left since you are adding a negative number to 1.

Draw the number line model in the space below.

Exercise 3

Find the following difference, and represent it on a number line. \(-5\frac{1}{2} - (-8)\).
Exercise 4

Find the following sums and differences using a number line model.

a. \(-6 + 5\frac{1}{4}\)

b. \(7 - (-0.9)\)

c. \(2.5 + (-\frac{1}{2})\)

d. \(-\frac{1}{4} + 4\)

e. \(\frac{1}{2} - (-3)\)
Exercise 5

Create an equation and number line diagram to model each answer.

a. Samantha owes her father $7. She just got paid $5.50 for babysitting. If she gives that money to her dad, how much will she still owe him?

b. At the start of a trip, a car’s gas tank contains 12 gallons of gasoline. During the trip, the car consumes $10 \frac{1}{8}$ gallons of gasoline. How much gasoline is left in the tank?

c. A fish was swimming $3 \frac{1}{2}$ feet below the water’s surface at 7:00 a.m. Four hours later, the fish was at a depth that is $5 \frac{1}{4}$ feet below where it was at 7:00 a.m. What rational number represents the position of the fish with respect to the water’s surface at 11:00 a.m.?
Lesson Summary

The rules for adding and subtracting integers apply to all rational numbers.

The sum of two rational numbers (e.g., \(-1 + 4.3\)) can be found on the number line by placing the tail of an arrow at \(-1\) and locating the head of the arrow 4.3 units to the right to arrive at the sum, which is 3.3.

To model the difference of two rational numbers on a number line (e.g., \(-5.7 - 3\)), first rewrite the difference as a sum, \(-5.7 + (-3)\), and then follow the steps for locating a sum. Place a single arrow with its tail at \(-5.7\) and the head of the arrow 3 units to the left to arrive at \(-8.7\).

Problem Set

Represent each of the following problems using both a number line diagram and an equation.

1. A bird that was perched atop a 15\( \frac{1}{2} \)foot tree dives down six feet to a branch below. How far above the ground is the bird’s new location?

2. Mariah had owed her grandfather $2.25 but was recently able to pay him back $1.50. How much does Mariah currently owe her grandfather?

3. Jake is hiking a trail that leads to the top of a canyon. The trail is 4.2 miles long, and Jake plans to stop for lunch after he completes 1.6 miles. How far from the top of the canyon will Jake be when he stops for lunch?

4. Sonji and her friend Rachel are competing in a running race. When Sonji is 0.4 miles from the finish line, she notices that her friend Rachel has fallen. If Sonji runs one tenth of a mile back to help her friend, how far will she be from the finish line?

5. Mr. Henderson did not realize his checking account had a balance of $200 when he used his debit card for a $317.25 purchase. What is his checking account balance after the purchase?

6. If the temperature is \(-3^\circ F\) at 10:00 p.m., and the temperature falls four degrees overnight, what is the resulting temperature?
Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Student Outcomes

- Students use properties of operations to add and subtract rational numbers without the use of a calculator.
- Students recognize that any problem involving addition and subtraction of rational numbers can be written as a problem using addition and subtraction of positive numbers only.
- Students use the commutative and associative properties of addition to rewrite numerical expressions in different forms. They know that the opposite of a sum is the sum of the opposites (e.g., \(- (3 + (-4)) = -3 + 4\)).

Lesson Notes

This lesson is the first of a two-day lesson using the properties of operations to add and subtract rational numbers. The lesson begins with a focus on representing the opposite of a sum as the sum of its opposites so that students may more efficiently arrive at sums and differences of rational numbers. The focus includes a representation of negative mixed numbers so that students conceptualize a negative mixed number as a negative integer plus a negative fraction.

Students often mistakenly add a negative mixed number to a positive whole number by adding the negative whole number part of the mixed number to the positive whole number but then erroneously representing the fractional part of the negative mixed number as a positive number.

The following is an example of the properties and how they are used in this lesson.

\[-13\frac{5}{7} + 6 - \frac{2}{7}\]

\[= -13\frac{5}{7} + 6 + \left(-\frac{2}{7}\right)\]

\[= -13 + \left(-\frac{5}{7}\right) + 6 + \left(-\frac{2}{7}\right)\]

\[= -13 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + 6\]

\[= -13 + (-1) + 6\]

\[= -14 + 6\]

\[= -8\]

Subtracting a number is the same as adding its inverse.

The opposite of a sum is the sum of its opposite.

Commutative property of addition

Associative property of addition
Classwork

Fluency Exercise (6 minutes): Integer Addition

Photocopy the attached 2-page fluency-building exercises so that each student receives a copy. Time the students, allowing one minute to complete Side A. Before students begin, inform them that they may not skip over questions and that they must move in order. After one minute, discuss the answers. Before administering Side B, elicit strategies from those students who were able to accurately complete many problems on Side A. Administer Side B in the same fashion, and review the answers. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Opening Exercise (3 minutes): Recall of a Number’s Opposite

This warm-up will prepare students for Exercise 1. Ahead of time, post a large number line on the side wall (either in poster form or with painter’s tape.)

As students enter the room, hand them a small sticky note with a rational number on it. Ask them to “find their opposite.” (Sticky notes will be such that each signed number has a “match” for opposite.) Students pair up according to opposites, walk to the number line on the side wall, and stick their numbers in the correct locations on the number line. The class comes to consensus that all numbers are placed in the correct location.

Example 1 (4 minutes): The Opposite of a Sum is the Sum of its Opposites

Have the following statement up on the board: “The opposite of a sum is the sum of its opposites.” Tell students we are going to use some numbers from the Opening Exercise to investigate this statement.

Ask two pairs of students (who were partners from the Opening Exercise) to come to the front of the room. (Choose students who had rational numbers that were integers, as they will be easier to understand in this example.) Have one person from each pair write their numbers on the board; let’s say they were 7 and −2. Then find the sum, 7 + (−2) = 5, and then find the opposite of the sum, −5. Now have their partners write their numbers on the board, −7 and 2, and then find the sum of these opposites, −5. Now we can see that the opposite of the sum is equal to the sum of the opposites.

```
<table>
<thead>
<tr>
<th>Rational Number</th>
<th>Rational Number</th>
<th>Sum</th>
<th>Opposite of the Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>−2</td>
<td>5</td>
<td>−5</td>
</tr>
<tr>
<td>Opposite Rational Number</td>
<td>Opposite Rational Number</td>
<td>Sum</td>
<td></td>
</tr>
<tr>
<td>−7</td>
<td>2</td>
<td>−5</td>
<td></td>
</tr>
</tbody>
</table>
```

It means that if you have a sum and want to take the opposite, for instance, −(7 + (−2)), you can rewrite it as the sum of each addend’s opposite −7 + 2.

Scaffolding:

- Select specific cards to give to students to challenge them at their level.

- Display an anchor poster in the classroom to show the meaning of “The opposite of a sum is the sum of its opposites.” Label the “opposite” and “sum” in a specific math example.
Exercise 1 (5 minutes)

Have students arrive at an answer to the following. Students share their different strategies with the class. The class members discuss the strategies they used. They determine which are most efficient, which ways are less likely to cause errors and confusion, whether they were able to reach the correct answer, etc. If no students share the solution method on the right, share it with the class.

Exercise 1

Represent the following expression with a single rational number.

$$-\frac{2}{5} + \frac{1}{4} - \frac{3}{5}$$

Two Possible Methods:

1. $$-\frac{8}{20} + \frac{5}{20} - \frac{12}{20}$$
2. $$-\frac{48}{20} + \frac{12}{20}$$

OR

1. $$-\frac{2}{5} + \frac{1}{4} + \left(-\frac{3}{5}\right)$$
2. $$-\frac{2}{5} + \left(-\frac{3}{5}\right) + \frac{3}{4}$$  \hspace{1cm} \text{commutative property}

After the students share their strategies, the following are questions that may guide the whole-group discussion. Student responses to the suggested discussion questions will vary.

- Was it difficult for you to add the mixed numbers with different signs and denominators? Why or why not?
- Were you able to arrive at the correct answer?
- Which method do you prefer?
- Which method is more challenging for you?

Example 2 (5 minutes): A Mixed Number Is a Sum

The following example allows students to focus on a mixed number as a sum. Looking at $2 \frac{2}{5}$, they think about how it can be rewritten using addition. (It means $2 + \frac{2}{5}$.) Once students represent it as a sum, they recognize that $-2 \frac{2}{5}$ means $-2 + \left(-\frac{2}{5}\right)$. The following is a possible lead-in question.

- $-2 \frac{2}{5}$ is the opposite of $2 \frac{2}{5}$. How can we show “the opposite of a sum is the sum of its opposites” with the number $-2 \frac{2}{5}$? How do we model it on a number line?
Example 2: A Mixed Number Is a Sum

Use the number line model shown below to explain and write the opposite of \(2 \frac{2}{5}\) as a sum of two rational numbers.

The opposite of a sum (top single arrow pointing left) and the sum of the opposites correspond to the sum point on the number line.

The opposite of \(2 \frac{2}{5}\) is \(-2 \frac{2}{5}\).

\(-2 \frac{2}{5}\) written as the sum of two rational numbers is \(-2 + \left(-\frac{2}{5}\right)\).

Exercise 2 (2 minutes)

Students independently rewrite each mixed number as the sum of two signed numbers. The teacher circulates the room providing assistance as needed. After two minutes, discuss the answers as a whole group.

Exercise 3 (2 minutes)

Students independently use the reverse process to represent each sum or difference as a mixed number. The teacher circulates the room providing assistance as needed. After two minutes, discuss the answers as a whole group.
Note: Exercises 3 and 4 are designed to provide students with an opportunity to practice writing mixed numbers as sums so they can do so as the need arises in more complicated problems.

**Exercise 4 (5 minutes)**

Students work independently to solve the problem below. Then student volunteers share their steps and solutions with the class. Note, the solution below includes just one possible solution method. However, a common mistake is for students to arrive at an incorrect answer of $-5 \frac{1}{8}$. As needed, revisit subtracting a mixed number from a whole number.

Mrs. Mitchell lost 10 pounds over the summer by jogging each week. By winter time, he had gained $5 \frac{1}{8}$ pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell’s weight?

$$-10 + \frac{1}{8}$$

$$= -10 + 5 + \frac{1}{8}$$

$$= (-10 + 5) + \frac{1}{8}$$

$$= (-5) + \frac{1}{8}$$

$$= -\frac{47}{8}$$

Mr. Mitchell’s weight dropped by $\frac{7}{8}$ pounds.

**Exercise 5 (7 minutes)**

Students work with a partner to complete the following exercise. Students make sense of each step and come up with an alternate method of solving the problem.

After five minutes, class resumes as a whole group, and students volunteer verbal explanations and their own methods for solving the problem.
Exercise 5

Jamal is completing a math problem and represents the expression \(-5 \frac{5}{7} + 8 - 3 \frac{2}{7}\) with a single rational number as shown in the steps below. Justify each of Jamal’s steps. Then, show another way to solve the problem.

\[
\begin{align*}
&= -5 \frac{5}{7} + 8 + \left(-2 \frac{2}{7}\right) \\
&= -5 \frac{5}{7} + \left(-3 \frac{2}{7}\right) + 8 \\
&= -5 + \left(-\frac{5}{7}\right) + (-3) + \left(-2 \frac{2}{7}\right) + 8 \\
&= -5 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + (-3) + 8 \\
&= -5 + (-1) + (-3) + 8 \\
&= -6 + (-3) + 8 \\
&= (-9) + 8 \\
&= -1
\end{align*}
\]

Step 1: Subtracting a number is the same as adding its inverse.

Step 2: Apply the commutative property of addition.

Step 3: The opposite of a sum is the sum of its opposites.

Step 4: Apply the commutative property of addition.

Step 5: Apply the associative property of addition.

\[
\left(-\frac{5}{7}\right) + \left(-2 \frac{2}{7}\right) = \left(-\frac{7}{7}\right) = -1
\]

Step 6: \(-5 + (-1) = -6\)

Step 7: \(-6 + (-3) = -9\)

Step 8: \(-9 + 8 = -1\)

Answers will vary for other methods of reaching a single rational number. Students may choose to add \(-5 \frac{5}{7}\) and 8 together first, but a common mistake is to represent their sum as \(3 \frac{3}{7}\) rather than \(2 \frac{2}{7}\).

Closing (2 minutes)

- How can we rewrite the opposite of a sum?
  - As the sum of its opposites.

- How is it helpful when finding the sums and differences of rational numbers to use the properties of operations?
  - It allows us to regroup terms so that we can efficiently arrive at an answer. For instance, in an expression we may wish to first combine certain rational numbers that are in decimal form or those that are in fractional form. Or, we may wish to group together all the negative numbers if we are finding the sum of positive and negative numbers.
Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,
  \[-\frac{5}{9} + 3.7 + \frac{2}{9} = \left(-\frac{5}{9} + \frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:
  \[-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right)
  \- (5 + 3) = -5 + (-3).

Exit Ticket (4 minutes)
Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Exit Ticket

Mariah and Shane both started to work on a math problem and were comparing their work in math class. Are both of their representations correct? Explain, and finish the math problem correctly to arrive at the correct answer.

Math Problem
Jessica’s friend lent her $5. Later that day Jessica gave her friend back $1 \frac{3}{4}$ dollars.

Which rational number represents the overall change to the amount of money Jessica’s friend has?

Mariah started the problem as follows:

\[-5 - \left( -1 \frac{3}{4} \right) = -5 + 1 - \frac{3}{4} \]

Shane started the problem as follows:

\[-5 - \left( -1 \frac{3}{4} \right) = -5 + \left( \frac{3}{4} \right) = -5 + \left( 1 + \frac{3}{4} \right) \]
Exit Ticket Sample Solutions

Mariah and Shane both started to work on a math problem and were comparing their work in math class. Are both of their representations correct? Explain, and finish the math problem correctly to arrive at the correct answer.

Math Problem
Jessica’s friend lent her $5. Later that day Jessica gave her friend back $1 \frac{3}{4} dollars.
Which rational number represents the overall change to the amount of money Jessica’s friend has?

Mariah started the problem as follows:

\[-5 - (-1 \frac{3}{4})\]
\[= -5 + 1 - \frac{3}{4}\]

Shane started the problem as follows:

\[-5 - (-1 \frac{3}{4})\]
\[= -5 + \left(1 + \frac{3}{4}\right)\]

Shane’s method is correct. In Mariah’s math work, she only dealt with part of the mixed number. The fractional part should have been positive too because the opposite of \(-1 \frac{3}{4}\) is \(1 \frac{3}{4}\) which contains both a positive 1 and a positive \(\frac{3}{4}\). The correct work would be

\[-5 - (-1 \frac{3}{4}) = -5 + \left(1 + \frac{3}{4}\right) = -5 + \left(\frac{3}{4}\right) = -5 + 1 + \frac{3}{4} = -4 + \frac{3}{4} = -3 \frac{1}{4}\]

The rational number would be \(-3 \frac{1}{4}\), which means Jessica’s friend gave away \(3 \frac{1}{4}\) dollars, or $3.25.

Problem Set Sample Solutions

1. Represent each sum as a single rational number.
   a. \(-14 + \left(-\frac{8}{9}\right)\)
   \[= -14 + \left(-\frac{8}{9}\right)\]
   \[= -\frac{126}{9} - \frac{8}{9}\]
   \[= -\frac{134}{9}\]
   
   b. \(7 + \frac{1}{9}\)
   \[= 7 + \frac{1}{9}\]
   \[= \frac{63}{9} + \frac{1}{9}\]
   \[= \frac{64}{9}\]
   
   c. \(-3 + \left(-\frac{1}{6}\right)\)
   \[= -3 + \left(-\frac{1}{6}\right)\]
   \[= -\frac{18}{6} - \frac{1}{6}\]
   \[= -\frac{19}{6}\]

   Rewrite each of the following to show that the opposite of a sum is the sum of the opposites. Problem 2 has been completed as an example.

2. \(-(9 + 8) = -9 + (-8)\)
   \[-17 = -17\]

   Answer provided in student materials.
3. \(- \left( \frac{1}{4} + 6 \right) = - \frac{1}{4} + (-6)\)
   \(-6 = -6\)

4. \(- \left( 10 + (-6) \right) = -10 + 6\)
   \(-4 = -4\)

5. \(- \left( \frac{55}{2} + \frac{1}{2} \right) = 55 + \left( -\frac{1}{2} \right)\)
   \(54 \frac{1}{2} = 54 \frac{1}{2}\)

Use your knowledge of rational numbers to answer the following questions.

6. Meghan said the opposite of the sum of \(-12\) and \(4\) is \(8\). Do you agree? Why or why not?
   Yes, I agree. The sum of \(-12\) and \(4\) is \(-8\), and the opposite of \(-8\) is \(8\).

7. Jolene lost her wallet at the mall. It had \(\$10\) in it. When she got home her brother felt sorry for her and gave her \(\$5.75\). Represent this situation with an expression involving rational numbers. What is the overall change in the amount of money Jolene has?
   \(-10 + 5.75 = -4.25\). The overall change in the amount of money Jolene has is \(-4.25\) dollars.

8. Isaiah is completing a math problem and is at the last step: \(25 - 28 \frac{1}{5}\). What is the answer? Show your work.
   \(25 - 28 \frac{1}{5} = 25 + \left( -28 + \left( -\frac{1}{5} \right) \right) = (25 + -28) + \left( -\frac{1}{5} \right) = -3 \frac{1}{5}\)

9. A number added to its opposite equals zero. What do you suppose is true about a sum added to its opposite?
   Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.

   A sum added to its opposite is zero.
   a. \((3 + 4) + (-3 + -4) = 7 + (-7) = 0\)
   b. \((-8 + 1) + (8 + (-1)) = (-7) + 7 = 0\)
   c. \(\left( -\frac{1}{2} + (-\frac{1}{4}) \right) + \left( \frac{1}{2} + \frac{1}{4} \right) = (-\frac{3}{4}) + \frac{3}{4} = 0\)
### Integer Addition – Round 1

**Directions:** Determine the sum of the integers, and write it in the column to the right.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>18.</td>
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<tr>
<td>2.</td>
<td>19.</td>
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<td>3.</td>
<td>20.</td>
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<td>4.</td>
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<td>15.</td>
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<td>16.</td>
<td>33.</td>
</tr>
<tr>
<td>17.</td>
<td>34.</td>
</tr>
</tbody>
</table>

- 8 + (−5)
- 10 + (−3)
- 2 + (−7)
- 4 + (−11)
- −3 + (−9)
- −12 + (−7)
- −13 + 5
- −4 + 9
- 7 + (−7)
- −13 + 13
- 14 + (−20)
- 6 + (−4)
- 10 + (−7)
- −16 + 9
- −10 + 34
- −20 + (−5)
- −18 + 15
- −38 + 25
- −19 + (−11)
- 2 + (−7)
- −23 + (−23)
- 45 + (−32)
- 16 + (−24)
- −28 + 13
- −15 + 15
- 12 + (−19)
- −24 + (−32)
- −18 + (−18)
- 14 + (−26)
- −7 + 8 + (−3)
- 2 + (−15) + 4
- −8 + (−19) + (−11)
- 15 + (−12) + 7
- −28 + 7 + (−7)

Number Correct: ______
### Integer Addition – Round 1 [KEY]

**Directions:** Determine the sum of the integers, and write it in the column to the right.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>8 + (−5)</td>
<td>3</td>
<td>18.</td>
<td>−38 + 25</td>
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<td>2.</td>
<td>10 + (−3)</td>
<td>7</td>
<td>19.</td>
<td>−19 + (−11)</td>
</tr>
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<td>3.</td>
<td>2 + (−7)</td>
<td>−5</td>
<td>20.</td>
<td>2 + (−7)</td>
</tr>
<tr>
<td>4.</td>
<td>4 + (−11)</td>
<td>−7</td>
<td>21.</td>
<td>−23 + (−23)</td>
</tr>
<tr>
<td>5.</td>
<td>−3 + (−9)</td>
<td>−12</td>
<td>22.</td>
<td>45 + (−32)</td>
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<td>6.</td>
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<td>−19</td>
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<td>16 + (−24)</td>
</tr>
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<td>7.</td>
<td>−13 + 5</td>
<td>−8</td>
<td>24.</td>
<td>−28 + 13</td>
</tr>
<tr>
<td>8.</td>
<td>−4 + 9</td>
<td>5</td>
<td>25.</td>
<td>−15 + 15</td>
</tr>
<tr>
<td>9.</td>
<td>7 + (−7)</td>
<td>0</td>
<td>26.</td>
<td>12 + (−19)</td>
</tr>
<tr>
<td>10.</td>
<td>−13 + 13</td>
<td>0</td>
<td>27.</td>
<td>−24 + (−32)</td>
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<tr>
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<td>−6</td>
<td>28.</td>
<td>−18 + (−18)</td>
</tr>
<tr>
<td>12.</td>
<td>6 + (−4)</td>
<td>2</td>
<td>29.</td>
<td>14 + (−26)</td>
</tr>
<tr>
<td>13.</td>
<td>10 + (−7)</td>
<td>3</td>
<td>30.</td>
<td>−7 + 8 + (−3)</td>
</tr>
<tr>
<td>14.</td>
<td>−16 + 9</td>
<td>−7</td>
<td>31.</td>
<td>2 + (−15) + 4</td>
</tr>
<tr>
<td>15.</td>
<td>−10 + 34</td>
<td>24</td>
<td>32.</td>
<td>−8 + (−19) + (−11)</td>
</tr>
<tr>
<td>16.</td>
<td>−20 + (−5)</td>
<td>−25</td>
<td>33.</td>
<td>15 + (−12) + 7</td>
</tr>
<tr>
<td>17.</td>
<td>−18 + 15</td>
<td>−3</td>
<td>34.</td>
<td>−28 + 7 + (−7)</td>
</tr>
</tbody>
</table>
## Integer Addition – Round 2

**Directions:** Determine the sum of the integers, and write it in the column to the right.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$5 + (-12)$</td>
</tr>
<tr>
<td>2.</td>
<td>$10 + (-6)$</td>
</tr>
<tr>
<td>3.</td>
<td>$-9 + (-13)$</td>
</tr>
<tr>
<td>4.</td>
<td>$-12 + 17$</td>
</tr>
<tr>
<td>5.</td>
<td>$-15 + 15$</td>
</tr>
<tr>
<td>6.</td>
<td>$16 + (-25)$</td>
</tr>
<tr>
<td>7.</td>
<td>$-12 + (-8)$</td>
</tr>
<tr>
<td>8.</td>
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</tr>
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<td>9.</td>
<td>$28 + (-12)$</td>
</tr>
<tr>
<td>10.</td>
<td>$-19 + (-19)$</td>
</tr>
<tr>
<td>11.</td>
<td>$-17 + 20$</td>
</tr>
<tr>
<td>12.</td>
<td>$8 + (-18)$</td>
</tr>
<tr>
<td>13.</td>
<td>$13 + (-15)$</td>
</tr>
<tr>
<td>14.</td>
<td>$-10 + (-16)$</td>
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<tr>
<td>15.</td>
<td>$35 + (-35)$</td>
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<tr>
<td>16.</td>
<td>$9 + (-14)$</td>
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<td>17.</td>
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<td>18.</td>
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<tr>
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<td>$-26 + (-19)$</td>
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<td>$16 + (-37)$</td>
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<td>21.</td>
<td>$-21 + 14$</td>
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<td>22.</td>
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<td>24.</td>
<td>$-16 + 16$</td>
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<tr>
<td>25.</td>
<td>$30 + (-43)$</td>
</tr>
<tr>
<td>26.</td>
<td>$-22 + (-18)$</td>
</tr>
<tr>
<td>27.</td>
<td>$-43 + 27$</td>
</tr>
<tr>
<td>28.</td>
<td>$38 + (-19)$</td>
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<tr>
<td>29.</td>
<td>$-13 + (-13)$</td>
</tr>
<tr>
<td>30.</td>
<td>$5 + (-8) + (-3)$</td>
</tr>
<tr>
<td>31.</td>
<td>$6 + (-11) + 14$</td>
</tr>
<tr>
<td>32.</td>
<td>$-17 + 5 + 19$</td>
</tr>
<tr>
<td>33.</td>
<td>$-16 + (-4) + (-7)$</td>
</tr>
<tr>
<td>34.</td>
<td>$8 + (-24) + 12$</td>
</tr>
</tbody>
</table>

Number Correct: ______

Improvement: ______
**Integer Addition – Round 2 [KEY]**

**Directions:** Determine the sum of the integers, and write it in the column to the right.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>$5 + (-12)$</td>
<td>$-7$</td>
</tr>
<tr>
<td>2.</td>
<td>$10 + (-6)$</td>
<td>$4$</td>
</tr>
<tr>
<td>3.</td>
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<td>$-22$</td>
</tr>
<tr>
<td>4.</td>
<td>$-12 + 17$</td>
<td>$5$</td>
</tr>
<tr>
<td>5.</td>
<td>$-15 + 15$</td>
<td>$0$</td>
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<td>6.</td>
<td>$16 + (-25)$</td>
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<td>8.</td>
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<td>11.</td>
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<td>$3$</td>
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<td>13.</td>
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<tr>
<td>15.</td>
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<td>$23 + (-31)$</td>
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<td>30.</td>
<td>$5 + (-8) + (-3)$</td>
<td>$-6$</td>
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<td>$6 + (-11) + 14$</td>
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<td>$-16 + (-4) + (-7)$</td>
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</tr>
<tr>
<td>34.</td>
<td>$8 + (-24) + 12$</td>
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</tr>
</tbody>
</table>
Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork

Example 1: The Opposite of a Sum is the Sum of its Opposites

Explain the meaning of: “The opposite of a sum is the sum of its opposites.” Use a specific math example.

<table>
<thead>
<tr>
<th>Rational Number</th>
<th>Rational Number</th>
<th>Sum</th>
<th>Opposite of the Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Opposite Rational Number</th>
<th>Opposite Rational Number</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercise 1

Represent the following expression with a single rational number.

\[-2 \frac{2}{5} + 3 \frac{1}{4} - 3 \frac{3}{5}\]
**Example 2: A Mixed Number is a Sum**

Use the number line model shown below to explain and write the opposite of $2\frac{2}{5}$ as a sum of two rational numbers.

![Number line diagram showing the opposite of a sum and the sum of the opposites]

The opposite of a sum (top single arrow pointing left) and the sum of the opposites correspond to the same point on the number line.

**Exercise 2**

Rewrite each mixed number as the sum of two signed numbers.

a. $-9\frac{5}{8}$  
b. $-2\frac{1}{2}$  
c. $8\frac{11}{12}$

**Exercise 3**

Represent each sum as a mixed number.

a. $-1 + (-\frac{5}{12})$  
b. $30 + \frac{1}{8}$  
c. $-17 + (-\frac{1}{9})$
Exercise 4

Mr. Mitchell lost 10 pounds over the summer by jogging each week. By winter time, he had gained 5 1/8 pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell’s weight?

Exercise 5

Jamal is completing a math problem and represents the expression \(-5 \frac{5}{7} + 8 - 3 \frac{2}{7}\) with a single rational number as shown in the steps below. Justify each of Jamal’s steps. Then, show another way to solve the problem.

\[
\begin{align*}
&= -5 \frac{5}{7} + 8 + \left( -3 \frac{2}{7} \right) \\
&= -5 \frac{5}{7} + \left( -3 \frac{2}{7} \right) + 8 \\
&= -5 + \left( -\frac{5}{7} \right) + (-3) + \left( -\frac{2}{7} \right) + 8 \\
&= -5 + \left( -\frac{5}{7} \right) + \left( -\frac{2}{7} \right) + (-3) + 8 \\
&= -5 + (-1) + (-3) + 8 \\
&= -6 + (-3) + 8 \\
&= (-9) + 8 \\
&= -1
\end{align*}
\]
Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,
  \[-\frac{2}{9} + 3.7 + 5 \frac{2}{9} = \left( -\frac{2}{9} + 5 \frac{2}{9} \right) + 3.7 = 0 + 3.7 = 3.7.\]
- The opposite of a sum is the sum of its opposites as shown in the examples that follow:
  \[-4\frac{4}{7} = -4 + \left( -\frac{4}{7} \right)\]
  \[-(5 + 3) = -5 + (-3)\]

Problem Set

1. Represent each sum as a single rational number.
   a. \(-14 + \left( -\frac{8}{9} \right)\)  
b. \(7 + \frac{1}{9}\)  
c. \(-3 + \left( -\frac{1}{6} \right)\)

Rewrite each of the following to show that the opposite of a sum is the sum of the opposites. Problem 2 has been completed as an example.

2. \(-(9 + 8) = -9 + (-8)\)
   \[-17 = -17\]

3. \(-\left( \frac{1}{4} + 6 \right)\)

4. \(-\left( 10 + (-6) \right)\)

5. \(-\left( (-55) + \frac{1}{2} \right)\)

Use your knowledge of rational numbers to answer the following questions.

6. Meghan said the opposite of the sum of \(-12\) and \(4\) is \(8\). Do you agree? Why or why not?

7. Jolene lost her wallet at the mall. It had \$10 in it. When she got home her brother felt sorry for her and gave her \$5.75. Represent this situation with an expression involving rational numbers. What is the overall change in the amount of money Jolene has?

8. Isaiah is completing a math problem and is at the last step: \(25 - 28 \frac{1}{5}\). What is the answer? Show your work.
9. A number added to its opposite equals zero. What do you suppose is true about a sum added to its opposite?
Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.

a. \((3 + 4) + (-3 + -4)\)
b. \((-8 + 1) + (8 + (-1))\)
c. \((-\frac{1}{2} + (-\frac{1}{4})) + (\frac{1}{2} + \frac{1}{4})\)
Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Student Outcomes

- Students use properties of operations to add and subtract rational numbers without the use of a calculator.
- Students recognize that any problem involving addition and subtraction of rational numbers can be written as a problem using addition and subtraction of positive numbers only.
- Students use the commutative and associative properties of addition to rewrite numerical expressions in different forms. They know that the opposite of a sum is the sum of the opposites; e.g., \((-3 - 4) = -3 + 4\).

Classwork

Fluency Exercise (6 minutes): Integer Subtraction

Photocopy the attached 2-page fluency-building exercises so that each student receives a copy. Time the students, allowing one minute to complete Side A. Before students begin, inform them that they may not skip over questions and that they must move in order. After one minute, discuss the answers. Before administering Side B, elicit strategies from those students who were able to accurately complete many problems on Side A. Administer Side B in the same fashion, and review the answers. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Exercise 1 (6 minutes)

Students are given scrambled steps to one possible solution to the following problem\(^1\). They work independently to arrange the expressions in an order that leads to a solution and record their solutions in the student materials.

Unscramble the cards, and show the steps in the correct order to arrive at the solution to \(\frac{2}{3} - (\frac{8}{1} + \frac{5}{9})\).

\[
\begin{align*}
0 + (-8.1) & \quad \left(\frac{5}{9} + (-\frac{5}{9})\right) + (-8.1) & \quad -8.1 \\
\frac{5}{9} + (-8.1 + (-\frac{5}{9})) & \quad \frac{5}{9} + (-\frac{5}{9} + (-8.1))
\end{align*}
\]

\(^1\) The scrambled steps may also be displayed on an interactive whiteboard, and students can come up one at a time to slide a step into the correct position.
After 2 minutes, students share the correct sequence of steps with the class.

- What allows us to represent operations in another form and rearrange the order of terms?
  - The properties of operations.
- Specifically which properties of operations were used in this example?
  - Students recall the additive inverse property and commutative property of addition. (Students are reminded to focus on all of the properties that justify their steps today.)
- Why did we use the properties of operations?
  - Students recognize that using the properties allows us to efficiently (more easily) calculate the answer to the problem.

**Examples 1–2 (7 minutes)**

Students record the following examples. Students assist in volunteering verbal explanations for each step during the whole-group discussion. Today, students’ focus is not on memorizing the names of each property but rather knowing that each representation is justifiable through the properties of operations.

**Examples 1–2**

Represent each of the following expressions as one rational number. Show and explain your steps.

1. \[ \frac{4}{7} - \left( \frac{4}{7} - 10 \right) \]

   \[ = \frac{4}{7} - \left( \frac{4}{7} + (-10) \right) \quad \text{Subtracting a number is the same as adding its inverse.} \]
   
   \[ = \frac{4}{7} + \left( \frac{-4}{7} + 10 \right) \quad \text{The opposite of a sum is the sum of its opposites.} \]
   
   \[ = \left( \frac{4}{7} + \frac{-4}{7} \right) + 10 \quad \text{The associative property of addition.} \]
   
   \[ = 0 + 10 \quad \text{A number plus its opposite equals zero.} \]
   
   \[ = 10 \]

- First, predict the answer. Explain your prediction.
  - Answers may vary.
The answer will be between 0 and \(\frac{1}{2}\) because \(5 + (-5) = 0\) and \(-4\frac{4}{7}\) is close to \(-5\), but 5 has a larger absolute value than \(-4\frac{4}{7}\). To add \(5 + (-4\frac{4}{7})\), we subtract their absolute values. Since \(-4\frac{4}{7}\) is close to \(-4\frac{1}{2}\), the answer will be about \(5 - 4\frac{1}{2} = \frac{1}{2}\).

2. \(5 + (-4\frac{4}{7})\)
   
   \[= 5 + \left[-\left(\frac{4}{7}\right)\right] \quad \text{The mixed number } 4\frac{4}{7} \text{ is equivalent to } 4 + \frac{4}{7}.\]
   \[= 5 + \left(-4 + \frac{4}{7}\right) \quad \text{The opposite of a sum is the sum of its opposites.}\]
   \[= (5 + (-4)) + \left(-\frac{4}{7}\right) \quad \text{Associative property of addition.}\]
   \[= 1 + \left(-\frac{4}{7}\right) \quad 5 + (-4) = 1\]
   \[= \frac{7}{7} + (-\frac{4}{7}) \quad \frac{7}{7} = 1\]
   \[= \frac{3}{7}\]

- Does our answer match our prediction?
  - Yes, we predicted a positive number close to zero.

Exercise 2 (10 minutes): Team Work!

Students work in groups of three. Each student has a different colored pencil. Each problem has at least three steps. Students take turns writing a step to each problem, passing the paper to the next person, and rotating the student who starts first with each new problem.

After 8 minutes, students partner up with another group of students to discuss or debate their answers. Students should also explain their steps and the properties/rules that justify each step.

Exercise 2: Team Work!

a. \(-5.2 - (-3.1) + 5.2\)
   
   \[= -5.2 + 3.1 + 5.2 \quad \text{MP.2 & MP.3}\]
   \[= -5.2 + 5.2 + 3.1 \quad \text{Associative property of addition.}\]
   \[= 0 + 3.1 \quad \text{MP.2 & MP.3}\]
   \[= 3.1\]

b. \(32 + (-12\frac{7}{8})\)
   
   \[= 32 + \left[-12 + \left(-\frac{7}{8}\right)\right] \quad \text{MP.2 & MP.3}\]
   \[= (32 + (-12)) + \left(-\frac{7}{8}\right) \quad \text{Associative property of addition.}\]
   \[= 20 + (-\frac{7}{8}) \quad \text{MP.2 & MP.3}\]
   \[= 19\frac{1}{8}\]
Exercise 3 (5 minutes)

Students work independently to answer the following question, then after 3 minutes, group members share their responses with one another and come to a consensus.

Exercise 3

Explain step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

\[-24 - \left(-\frac{1}{2}\right) - 12.5\]

**Subtracting \(-\frac{1}{2}\) is the same as adding its inverse \(\frac{1}{2}\):**

\[-24 + \frac{1}{2} + (-12.5)\]

**Next, I used the commutative property of addition to rewrite the expression:**

\[-24 + (-12.5) + \frac{1}{2}\]

**Next, I added both negative numbers:**

\[-36.5 + \frac{1}{2}\]

**Next, I wrote \(\frac{1}{2}\) in its decimal form:**

\[-36.5 + 0.5\]

**Lastly, I added \(-36.5 + 0.5\):**

\[-36\]

Closing (3 minutes)

- How are the properties of operations helpful when finding the sums and differences of rational numbers?
  - The properties of operations allow us to add and subtract rational numbers more efficiently.

- Do you think the properties of operations could be used in a similar way to aid in the multiplication and division of rational numbers?
  - Answers will vary.
Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,
  \[-\frac{5}{2} + 3.7 + \frac{5}{2} = \left(-\frac{5}{2} + \frac{5}{2}\right) + 3.7 = 0 + 3.7 = 3.7.\]

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:
  \[-4 \frac{4}{7} = -4 + \left(-\frac{4}{7}\right).\]
  \[-(5 + 3) = -5 + (-3).\]

Exit Ticket (8 minutes)
Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Exit Ticket

1. Jamie was working on his math homework with his friend, Kent. Jamie looked at the following problem.

\[-9.5 - (-8) - 6.5\]

He told Kent that he did not know how to subtract negative numbers. Kent said that he knew how to solve the problem using only addition. What did Kent mean by that? Explain. Then, show your work and represent the answer as a single rational number.

_______________________________________________________________________________________________
_______________________________________________________________________________________________
________________________________________
______________________________________________________

Work Space:

Answer: ______________________

2. Use one rational number to represent the following expression. Show your work.

\[3 + (-0.2) - 15 \frac{1}{4}\]
Exit Ticket Sample Solutions

1. Jamie was working on his math homework with his friend, Kent. Jamie looked at the following problem

$$-9.5 - (-8) - 6.5$$

He told Kent that he did not know how to subtract negative numbers. Kent said that he knew how to solve the problem using only addition. What did Kent mean by that? Explain. Then, show your work and represent the answer as a single rational number.

Kent meant that since any subtraction problem can be written as an addition problem by adding the opposite of the number you are subtracting, Jamie can solve the problem by using only addition.

Work Space:

$$-9.5 - (-8) - 6.5$$

$$= -9.5 + 8 + (-6.5)$$

$$= -9.5 + (-6.5) + 8$$

$$= -16 + 8$$

$$= -8$$

Answer: $$-8$$

2. Use one rational number to represent the following expression. Show your work.

$$3 + (-0.2) - 15 \frac{1}{4}$$

$$= 3 + (-0.2) + (-15 + (-\frac{1}{4}))$$

$$= 3 + (-0.2 + (-15) + (-0.25))$$

$$= 3 + (-15.45)$$

$$= -12.45$$

Problem Set Sample Solutions

Show all steps taken to rewrite each of the following as a single rational number.

1. $$80 + \left(-22 \frac{4}{15}\right)$$

$$= 80 + \left(-22 + \left(-\frac{4}{15}\right)\right)$$

$$= 80 + (-22) + \left(-\frac{4}{15}\right)$$

$$= 58 + \left(-\frac{4}{15}\right)$$

$$= 57 \frac{11}{15}$$

2. $$10 + \left(-3 \frac{3}{8}\right)$$

$$= 10 + \left(-3 + \left(-\frac{3}{8}\right)\right)$$

$$= (10 + (-3)) + \left(-\frac{3}{8}\right)$$

$$= 7 + \left(-\frac{3}{8}\right)$$

$$= 6 \frac{5}{8}$$
3. \[ \frac{1}{5} + 20.3 - \left( -\frac{3}{5} \right) \]
\[
= \frac{1}{5} + 20.3 + \frac{3}{5}
\]
\[
= \frac{1}{5} + 5\frac{3}{5} + 20.3
\]
\[
= 5\frac{4}{5} + 20.3
\]
\[
= 5\frac{1}{5} + 20 \frac{3}{10}
\]
\[
= 5\frac{8}{10} + 20 \frac{3}{10}
\]
\[
= 25 \frac{11}{10}
\]
\[
= 26 \frac{1}{10}
\]

4. \[ \frac{11}{12} - (-10) - \frac{5}{6} \]
\[
= \frac{11}{12} + 10 + \left( -\frac{5}{6} \right)
\]
\[
= \frac{11}{12} + \left( -\frac{5}{6} \right) + 10
\]
\[
= \frac{11}{12} + \left( -\frac{10}{12} \right) + 10
\]
\[
= 10 \frac{1}{12}
\]

5. Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

\[ 1 - \frac{3}{4} + (-12 \frac{1}{4}) \]

First, I rewrote the subtraction of \( \frac{3}{4} \) as the addition of its inverse \(-\frac{3}{4}\):  
\[
= 1 + \left( -\frac{3}{4} \right) + (-12 \frac{1}{4})
\]

Next, I used the associative property of addition to regroup addends:
\[
= 1 + \left( -\frac{3}{4} + (-12 \frac{1}{4}) \right)
\]

Next, I separated \(-12 \frac{1}{4}\) into the sum of \(-12\) and \(-\frac{1}{4}\):
\[
= 1 + \left( -\frac{3}{4} + (-12) + (-\frac{1}{4}) \right)
\]

Next, I used the commutative property of addition:
\[
= 1 + \left( -\frac{3}{4} + (-\frac{1}{4}) + (-12) \right)
\]

Next, I found the sum of \(-\frac{3}{4}\) and \(-\frac{1}{4}\):
\[
= 1 + (-\frac{1}{4}) + (-12)
\]

Next, I found the sum of \(-1\) and \(-12\):
\[
= 1 + (-13)
\]

Lastly, since the absolute value of 13 is greater than the absolute value of 1, and it is a negative 13, the answer will be a negative number. The absolute value of 13 minus the absolute value of 1 equals 12, so the answer is \(-12\).
### Integer Subtraction – Round 1

**Directions:** Determine the difference of the integers, and write it in the column to the right.

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### Integer Subtraction – Round 1 [KEY]

**Directions:** Determine the difference of the integers, and write it in the column to the right.

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### Integer Subtraction – Round 2

**Directions:** Determine the difference of the integers, and write it in the column to the right.

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<td>$(-35) - 9$</td>
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<tr>
<td>6.</td>
<td>$3 - 9$</td>
<td></td>
<td></td>
<td>28.</td>
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<tr>
<td>7.</td>
<td>$3 - 10$</td>
<td></td>
<td></td>
<td>29.</td>
<td>$(-27) - 27$</td>
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<tr>
<td>8.</td>
<td>$3 - 20$</td>
<td></td>
<td></td>
<td>30.</td>
<td>$(-14) - 21$</td>
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<tr>
<td>9.</td>
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<td>31.</td>
<td>$(-22) - 72$</td>
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<td>10.</td>
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<td></td>
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<td>32.</td>
<td>$(-311) - 611$</td>
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<tr>
<td>11.</td>
<td>$3 - (-1)$</td>
<td></td>
<td></td>
<td>33.</td>
<td>$(-345) - 654$</td>
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<tr>
<td>12.</td>
<td>$3 - (-2)$</td>
<td></td>
<td></td>
<td>34.</td>
<td>$(-2) - (-1)$</td>
<td></td>
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<tr>
<td>13.</td>
<td>$3 - (-3)$</td>
<td></td>
<td></td>
<td>35.</td>
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<td></td>
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<tr>
<td>14.</td>
<td>$3 - (-7)$</td>
<td></td>
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<td>36.</td>
<td>$(-2) - (-3)$</td>
<td></td>
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<tr>
<td>15.</td>
<td>$3 - (-17)$</td>
<td></td>
<td></td>
<td>37.</td>
<td>$(-2) - (-4)$</td>
<td></td>
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<tr>
<td>16.</td>
<td>$3 - (-27)$</td>
<td></td>
<td></td>
<td>38.</td>
<td>$(-2) - (-8)$</td>
<td></td>
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<tr>
<td>17.</td>
<td>$3 - (-127)$</td>
<td></td>
<td></td>
<td>39.</td>
<td>$(-20) - (-45)$</td>
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<tr>
<td>18.</td>
<td>$13 - (-6)$</td>
<td></td>
<td></td>
<td>40.</td>
<td>$(-24) - (-13)$</td>
<td></td>
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<tr>
<td>19.</td>
<td>$24 - (-8)$</td>
<td></td>
<td></td>
<td>41.</td>
<td>$(-13) - (-24)$</td>
<td></td>
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<tr>
<td>20.</td>
<td>$5 - (-23)$</td>
<td></td>
<td></td>
<td>42.</td>
<td>$(-5) - (-3)$</td>
<td></td>
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<tr>
<td>21.</td>
<td>$61 - (-3)$</td>
<td></td>
<td></td>
<td>43.</td>
<td>$(-3) - (-5)$</td>
<td></td>
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<tr>
<td>22.</td>
<td>$58 - (-5)$</td>
<td></td>
<td></td>
<td>44.</td>
<td>$(-1,034) - (-31)$</td>
<td></td>
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</tbody>
</table>

**Number Correct:** ______

**Improvement:** ______
**Lesson 9**: Applying the Properties of Operations to Add and Subtract Rational Numbers

### Integer Subtraction – Round 2 [KEY]

**Directions**: Determine the difference of the integers, and write it in the column to the right.

|   | 3 \(-\) 2 |   | 3 \(-\) 3 |   | 3 \(-\) 4 |   | 3 \(-\) 5 |   | 3 \(-\) 6 |   | 3 \(-\) 9 |   | 3 \(-\) 10 |   | 3 \(-\) 20 |   | 3 \(-\) 80 |   | 3 \(-\) 100 |   | 3 \(-\) \((-1)\) |   | 3 \(-\) \((-2)\) |   | 3 \(-\) \((-3)\) |   | 3 \(-\) \((-7)\) |   | 3 \(-\) \((-17)\) |   | 3 \(-\) \((-27)\) |   | 3 \(-\) \((-127)\) |   | 13 \(-\) \((-6)\) |   | 24 \(-\) \((-8)\) |   | 5 \(-\) \((-23)\) |   | 61 \(-\) \((-3)\) |   | 58 \(-\) \((-5)\) |   | 23 \((-8)\) \(-\) 5 |   | 24 \((-8)\) \(-\) 7 |   | 25 \((-8)\) \(-\) 9 |   | 26 \((-15)\) \(-\) 9 |   | 27 \((-35)\) \(-\) 9 |   | 28 \((-22)\) \(-\) 22 |   | 29 \((-27)\) \(-\) 27 |   | 30 \((-14)\) \(-\) 21 |   | 31 \((-22)\) \(-\) 72 |   | 32 \((-311)\) \(-\) 611 |   | 33 \((-345)\) \(-\) 654 |   | 34 \((-2)\) \(-\) \((-1)\) |   | 35 \((-2)\) \(-\) \((-2)\) |   | 36 \((-2)\) \(-\) \((-3)\) |   | 37 \((-2)\) \(-\) \((-4)\) |   | 38 \((-2)\) \(-\) \((-8)\) |   | 39 \((-20)\) \(-\) \((-45)\) |   | 40 \((-24)\) \(-\) \((-13)\) |   | 41 \((-13)\) \(-\) \((-24)\) |   | 42 \((-5)\) \(-\) \((-3)\) |   | 43 \((-3)\) \(-\) \((-5)\) |   | 44 \((-1,034)\) \(-\) \((-31)\) |   |
| 1 | 1 | 2 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 10 | 20 | 30 | 130 | 19 | 32 | 28 | 64 | 63 | -13 | -15 | -17 | -24 | -44 | -44 | -54 | -35 | -94 | -922 | -999 | -1 | 0 | 1 | 2 | 6 | 25 | -11 | 11 | -2 | 2 | -1,003 |
Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork
Exercise 1
Unscramble the cards, and show the steps in the correct order to arrive at the solution to $\left(\frac{5}{9} - \left(8.1 + \frac{2}{9}\right)\right)$.

\[
\begin{align*}
0 + (-8.1) \\
\left(\frac{5}{9} + \left(-\frac{2}{9}\right)\right) + (-8.1) \\
-8.1 \\
\frac{5}{9} + \left(-8.1 + \left(-\frac{2}{9}\right)\right) \\
\frac{5}{9} + \left(-\frac{2}{9} + (-8.1)\right)
\end{align*}
\]
Examples 1–2

Represent each of the following expressions as one rational number. Show and explain your steps.

1. \[4 \frac{4}{7} - \left(4 \frac{4}{7} - 10\right)\]

2. \[5 + \left(-4 \frac{4}{7}\right)\]
Exercise 2: Team Work!

a. $-5.2 - (-3.1) + 5.2$

b. $32 + \left(-12 \frac{7}{8}\right)$

c. $3 \frac{1}{6} + 20.3 - \left(-5 \frac{5}{6}\right)$

d. $\frac{16}{20} - (-1.8) - \frac{4}{5}$

Exercise 3

Explain step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$-24 - \left(-\frac{1}{2}\right) - 12.5$$
Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,
  \[ -5 \frac{2}{9} + 3.7 + 5 \frac{2}{9} = (-5 \frac{2}{9} + 5 \frac{2}{9}) + 3.7 = 0 + 3.7 = 3.7. \]
- The opposite of a sum is the sum of its opposites as shown in the examples that follow:
  \[ -4 \frac{4}{7} = -4 + \left( -\frac{4}{7} \right). \]
  \[ -(5 + 3) = -5 + (-3). \]

Problem Set

Show all steps taken to rewrite each of the following as a single rational number.

1. \[ 80 + \left( -22 \frac{4}{15} \right) \]

2. \[ 10 + \left( -3 \frac{3}{8} \right) \]

3. \[ \frac{1}{5} + 20.3 - \left( -5 \frac{3}{7} \right) \]

4. \[ \frac{11}{12} - (-10) - \frac{5}{6} \]

5. Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

\[ 1 - \frac{3}{4} + \left( -12 \frac{1}{4} \right) \]