Lesson 4: Increasing and Decreasing Functions

Classwork

Graphs are useful tools in terms of representing data. They provide a visual story, highlighting important facts that surround the relationship between quantities.

The graph of a linear function is a line. The slope of the line can provide useful information about the functional relationship between the two types of quantities:

- A linear function whose graph has a positive slope is said to be an increasing function.
- A linear function whose graph has a negative slope is said to be a decreasing function.
- A linear function whose graph has a zero slope is said to be a constant function.

Exercises

1. Read through each of the scenarios and choose the graph of the function that best matches the situation. Explain the reason behind each choice.
   a. A bathtub is filled at a constant rate of 1.75 gallons per minute.
   b. A bathtub is drained at a constant rate of 2.5 gallons per minute.
   c. A bathtub contains 2.5 gallons of water.
   d. A bathtub is filled at a constant rate of 2.5 gallons per minute.

<table>
<thead>
<tr>
<th>Scenario:</th>
<th>Explanation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (minutes)</td>
<td>Amount of Water in Tub (gallons)</td>
</tr>
</tbody>
</table>
2. Read through each of the scenarios, and sketch a graph of a function that models the situation.
   
a. A messenger service charges a flat rate of $4.95 to deliver a package regardless of distance to the destination.

   ![Graph of Delivery Charge vs Distance]

   b. At sea level, the air that surrounds us presses down on our bodies at 14.7 pounds per square inch (psi). For every 10 meters that you dive under water, the pressure increases by 14.7 psi.

   ![Graph of Pressure vs Depth]

c. The range (driving distance per charge) of an electric car varies based on the average speed the car is driven. The initial range of the electric car after a full charge is 400 miles. However, the range is reduced by 20 miles for every 10 mph increase in average speed the car is driven.

3. The graph below represents the total number of smart phones that are shipped to a retail store over the course of 50 days.

Match each part of the graph (A, B, and C) to its verbal description. Explain the reasoning behind your choice.

i. Half of the factory workers went on strike, and not enough smartphones were produced for normal shipments.
ii. The production schedule was normal, and smartphones were shipped to the retail store at a constant rate.

iii. A defective electronic chip was found, and the factory had to shut down; so, no smartphones were shipped.

4. The relationship between Jameson’s account balance and time is modeled by the graph below.

a. Write a story that models the situation represented by the graph.

b. When is the function represented by the graph increasing? How does this relate to your story?

c. When is the function represented by the graph decreasing? How does this relate to your story?
Lesson Summary

The graph of a function can be used to help describe the relationship between two quantities. The slope of the line can provide useful information about the functional relationship between two quantities:

- A function whose graph has a positive slope is said to be an increasing function.
- A function whose graph has a negative slope is said to be a decreasing function.
- A function whose graph has a zero slope is said to be a constant function.

Problem Set

1. Read through each of the scenarios, and choose the graph of the function that best matches the situation. Explain the reason behind each choice.
   a. The tire pressure on Regina’s car remains at 30 psi.
   b. Carlita inflates her tire at a constant rate for 4 minutes.
   c. Air is leaking from Courtney’s tire at a constant rate.
Lesson 4: Increasing and Decreasing Functions

Scenario:

Explanation:

Scenario:

Explanation:
2. A home was purchased for $275,000. Due to a recession, the value of the home fell at a constant rate over the next 5 years.
   a. Sketch a graph of a function that models the situation.

   ![Graph](image)

   b. Based on your graph, how is the home value changing with respect to time?

3. The graph below displays the first hour of Sam’s bike ride.

   ![Graph](image)

   Match each part of the graph (A, B, and C) to its verbal description. Explain the reasoning behind your choice.
   i. Sam rides his bike to his friend’s house at a constant rate.
   ii. Sam and his friend bike together to an ice cream shop that is between their houses.
   iii. Sam plays at his friend’s house.
4. Using the axes below, create a story about the relationship between two quantities.
   a. Write a story about the relationship between two quantities. Any quantities can be used (e.g., distance and time, money and hours, age and growth). Be creative! Include keywords in your story such as *increase* and *decrease* to describe the relationship.
   b. Label each axis with the quantities of your choice, and sketch a graph of the function that models the relationship described in the story.
Lesson 5: Increasing and Decreasing Functions

Classwork

Example 1: Nonlinear Functions in the Real World

Not all real-world situations can be modeled by a linear function. There are times when a nonlinear function is needed to describe the relationship between two types of quantities. Compare the two scenarios:

a. Aleph is running at a constant rate on a flat paved road. The graph below represents the total distance he covers with respect to time.

![Graph of a linear function showing constant rate]

b. Shannon is running on a flat, rocky trail that eventually rises up a steep mountain. The graph below represents the total distance she covers with respect to time.

![Graph of a nonlinear function showing increasing distance]

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Exercises 1–2

1. In your own words, describe what is happening as Aleph is running during the following intervals of time.
   a. 0 to 15 minutes
   b. 15 to 30 minutes
   c. 30 to 45 minutes
   d. 45 to 60 minutes

2. In your own words, describe what is happening as Shannon is running during the following intervals of time.
   a. 0 to 15 minutes
   b. 15 to 30 minutes
   c. 30 to 45 minutes
   d. 45 to 60 minutes
Example 2: Increasing and Decreasing Functions

The rate of change of a function can provide useful information about the relationship between two quantities. A linear function has a constant rate of change. A nonlinear function has a variable rate of change.

<table>
<thead>
<tr>
<th>Linear Functions</th>
<th>Nonlinear Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear function <em>increasing</em> at a constant rate</td>
<td>Nonlinear function <em>increasing</em> at a variable rate</td>
</tr>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Linear function <em>decreasing</em> at a constant rate</td>
<td>Nonlinear function <em>decreasing</em> at a variable rate</td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td>Linear function with a constant rate</td>
<td>Nonlinear function with a variable rate</td>
</tr>
<tr>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
<tr>
<td>(x)</td>
<td>(y)</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
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<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
</tr>
</tbody>
</table>
Exercises 3–5

3. Different breeds of dogs have different growth rates. A large breed dog typically experiences a rapid growth rate from birth to age 6 months. At that point, the growth rate begins to slow down until the dog reaches full growth around 2 years of age.

   a. Sketch a graph that represents the weight of a large breed dog from birth to 2 years of age.

   ![Graph of dog weight growth]

   b. Is the function represented by the graph linear or nonlinear? Explain.

   c. Is the function represented by the graph increasing or decreasing? Explain.
4. Nikka took her laptop to school and drained the battery while typing a research paper. When she returned home, Nikka connected her laptop to a power source, and the battery recharged at a constant rate.
   a. Sketch a graph that represents the battery charge with respect to time.
   ![Graph of Battery Charge vs. Time]

   b. Is the function represented by the graph linear or nonlinear? Explain.

   c. Is the function represented by the graph increasing or decreasing? Explain.
5. The long jump is a track and field event where an athlete attempts to leap as far as possible from a given point. Mike Powell of the United States set the long jump world record of 8.95 meters (29.4 feet) during the 1991 World Championships in Tokyo, Japan.

a. Sketch a graph that represents the path of a high school athlete attempting the long jump.

b. Is the function represented by the graph linear or nonlinear? Explain.

c. Is the function represented by the graph increasing or decreasing? Explain.
Example 3: Ferris Wheel

Lamar and his sister are riding a Ferris wheel at a state fair. Using their watches, they find that it takes 8 seconds for the Ferris wheel to make a complete revolution. The graph below represents Lamar and his sister’s distance above the ground with respect to time.

Exercises 6–9

6. Use the graph from Example 3 to answer the following questions.
   a. Is the function represented by the graph linear or nonlinear?
   b. Where is the function increasing? What does this mean within the context of the problem?
   c. Where is the function decreasing? What does this mean within the context of the problem?
7. How high above the ground is the platform for passengers to get on the Ferris wheel? Explain your reasoning.

8. Based on the graph, how many revolutions does the Ferris wheel complete during the 40 second time interval? Explain your reasoning.

9. What is the diameter of the Ferris wheel? Explain your reasoning.
Lesson Summary

The graph of a function can be used to help describe the relationship between two quantities.

A linear function has a constant rate of change. A nonlinear function does not have a constant rate of change.

- A function whose graph has a positive rate of change is an increasing function.
- A function whose graph has a negative rate of change is a decreasing function.
- Some functions may increase and decrease over different intervals.

Problem Set

1. Read through the following scenarios and match each to its graph. Explain the reasoning behind your choice.
   a. This shows the change in a smartphone battery charge as a person uses the phone more frequently.
   b. A child takes a ride on a swing.
   c. A savings account earns simple interest at a constant rate.
   d. A baseball has been hit at a little league game.
2. The graph below shows the volume of water for a given creek bed during a 24-hour period. On this particular day, there was wet weather with a period of heavy rain.

Describe how each part (A, B, and C) of the graph relates to the scenario.
3. Half-life is the time required for a quantity to fall to half of its value measured at the beginning of the time period. If there are 100 grams of a radioactive element to begin with, there will be 50 grams after the first half-life, 25 grams after the second half-life, and so on.
   a. Sketch a graph that represents the amount of the radioactive element left with respect to the number of half-lives that have passed.

   ![Graph](image)

   b. Is the function represented by the graph linear or nonlinear? Explain.
   c. Is the function represented by the graph increasing or decreasing?

4. Lanae parked her car in a No Parking zone. Consequently, her car was towed to an impound lot. In order to release her car, she needs to pay the impound lot charges. There is an initial charge on the day the car is brought to the lot. However, 10% of the previous day’s charges will be added to the total charge for every day the car remains in the lot.
   a. Sketch a graph that represents the total charges with respect to the number of days a car remains in the impound lot.

   ![Graph](image)
5. Kern won a $50 gift card to his favorite coffee shop. Every time he visits the shop, he purchases the same coffee drink.
   a. Sketch a graph of a function that can be used to represent the amount of money that remains on the gift card with respect to the number of drinks purchased.

   ![Graph](image)

   b. Is the function represented by the graph linear or nonlinear? Explain.
   c. Is the function represented by the graph increasing or decreasing? Explain.

   6. Jay and Brooke are racing on bikes to a park 8 miles away. The tables below display the total distance each person biked with respect to time.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Distance (miles)</th>
<th>Time (minutes)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.84</td>
<td>5</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>1.86</td>
<td>10</td>
<td>2.4</td>
</tr>
<tr>
<td>15</td>
<td>3.00</td>
<td>15</td>
<td>3.6</td>
</tr>
<tr>
<td>20</td>
<td>4.27</td>
<td>20</td>
<td>4.8</td>
</tr>
<tr>
<td>25</td>
<td>5.67</td>
<td>25</td>
<td>6.0</td>
</tr>
</tbody>
</table>

   a. Which person’s biking distance could be modeled by a nonlinear function? Explain.
   b. Who would you expect to win the race? Explain.
7. Using the axes below, create a story about the relationship between two quantities.
   a. Write a story about the relationship between two quantities. Any quantities can be used (e.g., distance and
time, money and hours, age and growth). Be creative! Include keywords in your story such as *increase* and
*decrease* to describe the relationship.
   b. Label each axis with the quantities of your choice, and sketch a graph of the function that models the
relationship described in the story.