

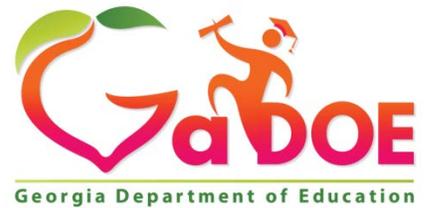


Georgia Standards of Excellence Comprehensive Course Overview

Mathematics

GSE Grade 6

2015-2016



Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Georgia Department of Education

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***Revised standards, tasks, and other resource indicated in bold red font.**

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Mathematics | Grade 6 | Curriculum Map

GSE Grade 6 Curriculum Map							
1 st Semester				2 nd Semester			
Unit 1 (4 – 5 weeks)	Unit 2 (3 – 4 weeks)	Unit 3 (4 – 5 weeks)	Unit 4 (4 – 5 weeks)	Unit 5 (4 – 5 weeks)	Unit 6 (4 – 5 weeks)	Unit 7 (3 – 4 weeks)	Unit 8 (3 – 4 weeks)
Number System Fluency	Rate, Ratio and Proportional Reasoning Using Equivalent	Expressions	One-Step Equations and Inequalities	Area and Volume	Statistics	Rational Explorations: Numbers and their Opposites	Show What We Know
MGSE6.NS.1 MGSE6.NS.2 MGSE6.NS.3 MGSE6.NS.4	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3a MGSE6.RP.3b MGSE6.RP.3c MGSE6.RP.3d	MGSE6.EE.1 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c MGSE6.EE.3 MGSE6.EE.4 MGSE6.NS.4	MGSE6.EE.5 MGSE6.EE.6 MGSE6.EE.7 MGSE6.EE.8 MGSE6.EE.9 MGSE6.RP.3a MGSE6.RP.3b MGSE6.RP.3c MGSE6.RP.3d (equations)	MGSE6.G.1 MGSE6.G.2 MGSE6.G.4	MGSE6.SP.1 MGSE6.SP.2 MGSE6.SP.3 MGSE6.SP.4 MGSE6.SP.5	MGSE6.NS.5 MGSE6.NS.6a MGSE6.NS.6b MGSE6.NS.6c MGSE6.NS.7a MGSE6.NS.7b MGSE6.NS.7c MGSE6.NS.7d MGSE6.NS.8 MGSE6.G.3	ALL
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain. *Revised standards indicated in bold red font.							

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 6-8 Key:

- NS = The Number System
- RP = Ratios and Proportional Relationships
- EE = Expressions and Equations
- G = Geometry
- SP = Statistics and Probability

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The Comprehensive Course Overviews are designed to provide access to multiple sources of support for implementing and instructing courses involving the Georgia Standards of Excellence.

GSE Mathematics Grade 6 Critical Areas

The middle school standards specify the mathematics that all students should study in order to be high school ready. The middle school standards are listed in conceptual categories including Number Sense, Algebra, Expressions and Equations, Geometry, and Statistics and Probability.

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking. Descriptions of the four critical areas follow:

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

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(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

GSE Mathematics Grade 6 Unit Descriptions

The sixth grade standards are arranged into units that will extend their knowledge and understanding of elementary topics into increasingly formalized and applicable skills as they transition into upper grades. The Standards for Mathematical Practice are a key component as they are applied in each course to equip students in making sense of problems and building a set of tools they can use in real-world situations.

Rather than racing to cover many topics in a “mile-wide, inch-deep curriculum”, the standards ask mathematics teachers to significantly narrow and deepen the way time and energy are spent in the classroom. The elementary grades focused on concepts, skills, and problem solving related to addition and subtraction, multiplication and division of whole numbers, and fractions.

In grade 6, the focus is on ratios and proportional relationships, and early algebraic expressions and equations.

Unit 1: By the end of fifth grade, students have had a variety of experiences working with whole numbers and fractions. In the first unit, work with whole numbers continues into dividing multi-digit numbers using the standard algorithm. All four operations with decimals, as well as dividing fractions by fractions, are emphasized from a hands-on approach in order to build understanding, not rely on memorization of rules and procedures. Students also find common factors and multiples as they progress in their understanding of composition and decomposition of numbers and become fluent in number sense.

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Unit 2: Ratios and rational relationships form an important undergirding of the entire sixth grade mathematics curriculum. Understanding ratio and “rational thinking” is critical to all future mathematics courses, and from the second unit throughout the year students revisit and continue to use the skills developed in this unit as they explore other topics. Their work with ratios includes unit rate and using rate to solve real-world problems.

Unit 3: The formal study of algebra begins in earnest in sixth grade, as students move from arithmetic understandings to algebraic expressions. Students learn to translate verbal phrases into algebraic expressions and utilize exponential notation in appropriate situations.

Unit 4: Extending the work begun in Unit 3, Unit 4 has students reason about and solve one-variable equations and inequalities. Mathematics is all about answering questions, finding the solutions to unknowns, and making sense of real-life situations. Students also learn that often two things are not balanced or equal, but are unequal, and they explore inequalities using tools such as number lines to become fluent in grasping the magnitude of numbers.

Unit 5: The study of geometry is interesting and fun for many students, as it is often more concrete and visual than some other domains of mathematics. Sixth grade students extend their understanding of the meaning of area and volume from elementary grades, now often having fractional edge lengths to work with instead of only whole number lengths. This represents the types of measurements they very often encounter in real-life, and helps students understand magnitude and applications of operations on fractional numbers.

Additionally, the fifth unit has students find area by composing and decomposing figures into familiar shapes, triangles and rectangles. They also use nets of three-dimensional figures to find surface area.

Unit 6: Sixth grade provides the first formal introduction to the study of statistics. Students begin by learning what questions are statistical in nature. That is, they are questions which will generate a range of responses. Unit 6 introduces the idea that data can be collected to answer a statistical question, then described by its center, spread, and overall shape. Statistical measures allow the description of a set of data and the spread of the data in single number summary, and tasks in Unit 6 acquaint students with this new domain.

Unit 7: Up to this point, students have only encountered numbers with values greater than or equal to zero (Natural Numbers, Counting Numbers, Whole Numbers). Unit 7 introduces students conceptually to circumstances best described with *negative* numbers, numbers with a *value less than zero*- the set of Integers. Operations with Integers are deliberately postponed to seventh grade, but by introducing students to Integers in sixth grade, they have the opportunity to explore situations appropriately represented by negative numbers, and graph points in all four quadrants of the coordinate plane. Using a number line, students learn about numbers and their “opposites”, and absolute value (distance from zero). This unit is intentionally at the end of sixth grade, as students are NOT expected to do any operations with Integers. Instead, this unit is to be an introduction. It leads directly into the first seventh grade unit, Operations with Rational Numbers.

Flipbooks

These “FlipBooks” were developed by the Kansas Association of Teachers of Mathematics (KATM) and are a compilation of research, “unpacked” standards from many states, instructional strategies and examples for each standard at each grade level. The intent is to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. The [Grade 6 Flipbook](#) is an interactive document arranged by the content domains listed on the following pages. The links on each domain and standard will take you to specific information on that standard/domain within the Flipbook.

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed. *For example, If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. For example, given $1 \text{ in.} = 2.54 \text{ cm}$, how many centimeters are in 6 inches?

The Number System

6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.

For example:

- *Create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.)*
- *How much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally?*
- *How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt?*
- *How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

- a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) Example: $36 + 8 = 4(9 + 2)$**
- b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.**

Apply and extend previous understandings of numbers to the system of rational numbers.

MGSE6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

MGSE6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

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MGSE6.NS.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

MGSE6.NS.7 Understand ordering and absolute value of rational numbers.

MGSE6.NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*

MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express the fact that -3°C is warmer than -7°C .*

MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.*

MGSE6.NS.7d Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*

MGSE6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations**6.EE****Apply and extend previous understandings of arithmetic to algebraic expressions.**

MGSE6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract y from 5” as $5 - y$.*

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.*

MGSE6.EE.2c Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

MGSE6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

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MGSE6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

- a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
- b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Solve real-world and mathematical problems involving area, surface area, and volume.

MGSE6.G.1 Find area of right triangles, other triangles, quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

MGSE6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths ($\frac{1}{2}$ u), and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = (\text{length}) \times (\text{width}) \times (\text{height})$ and $V = (\text{area of base}) \times (\text{height})$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

MGSE6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

MGSE6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Develop understanding of statistical variability.

MGSE6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*

MGSE6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

MGSE6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

MGSE6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

MGSE6.SP.5 Summarize numerical data sets in relation to their context, such as by:

- a. Reporting the number of observations.**
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.**
- c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range).**
- d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data was gathered.**

GSE Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The BLUE links will provide access to classroom videos on each standard for mathematical practice accessed on the [Inside Math](#) website.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”

2 Reason abstractly and quantitatively.

In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3 Construct viable arguments and critique the reasoning of others.

In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others' thinking.

4 Model with mathematics.

In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and

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explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5 Use appropriate tools strategically.

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.

6 Attend to precision.

In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.

7 Look for and make use of structure.

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality), $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.

8 Look for and express regularity in repeated reasoning.

In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Connecting the Standards for Mathematical Practice to the Content Standards

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous

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problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Classroom Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards based classroom and will support students' performances on the tasks in this unit and throughout the school year.

Strategies for Teaching and Learning

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
 - What level of support do my struggling students need in order to be successful with this unit?
 - In what way can I deepen the understanding of those students who are competent in this unit?

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- What real life connections can I make that will help my students utilize the skills practiced in this unit?

Tasks

The framework tasks represent the level of depth, rigor, and complexity expected of all sixth grade students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task). The table below provides a brief explanation of the types of tasks that teachers will find in the frameworks units for Coordinate Algebra.

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Short Cycle Task	Designed to exemplify the performance targets that the standards imply. The tasks, with the associated guidance, equip teachers to monitor overall progress in their students' mathematics.
Formative Assessment Lesson (FAL) *more information on page 20	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
3-Act Task *more information on page 21	A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
Achieve CCSS-CTE Classroom Tasks	Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

Formative Assessment Lessons (FALs)

What is a Formative Assessment Lesson (FAL)? The Formative Assessment Lesson is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to **inform** the instruction that will take place for the remainder of the unit.

Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

What does a Formative Assessment Lesson look like in action? Videos of Georgia Teachers implementing FALs can be accessed [HERE](#) and a sample of a FAL lesson may be seen [HERE](#)

Where can I find more information on FALs? More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the [Math Assessment Project](#)'s guide for teachers.

Where can I find samples of FALs?

Formative Assessment Lessons can also be found at the following sites:

[Mathematics Assessment Project](#)

[Kenton County Math Design Collaborative](#)

[MARS Tasks by grade level](#)

A **sample FAL** with extensive dialog and suggestions for teachers may be found [HERE](#). This resource will help teachers understand the flow and purpose of a FAL.

Where can I find more training on the use of FALs? The Math Assessment Project has developed Professional Development Modules that are designed to help teachers with the practical and pedagogical challenges presented by these lessons.

[Module 1](#) introduces the model of *formative assessment* used in the lessons, its theoretical background and practical implementation. [Modules 2 & 3](#) look at the two types of *Classroom Challenges* in detail. [Modules 4 & 5](#) explore two crucial pedagogical features of the lessons: asking probing questions and collaborative learning.

Georgia RESA's may be contacted about professional development on the use of FALs in the classroom. The request should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC).

Spotlight Tasks

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-Act Tasks

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

Guidelines for 3-Act Tasks and Patient Problem Solving (Teaching without the Textbook)

Adapted from Dan Meyer

Developing the mathematical Big Idea behind the 3-Act task:

- Create or find/use a clear visual which tells a brief, perplexing mathematical story. Video or live action works best. (See resource suggestions in the Guide to 3-Act Tasks)
- Video/visual should be real life and allow students to see the situation unfolding.
- Remove the initial literacy/mathematics concerns. Make as few language and/or math demands on students as possible. You are posing a mathematical question without words.
- The visual/video should inspire curiosity or perplexity which will be resolved via the mathematical big idea(s) used by students to answer their questions. You are creating an intellectual need or cognitive dissonance in students.

Enacting the 3-Act in the Classroom

Act 1 (The Question):

Set up student curiosity by sharing a scenario:

- Teacher says, “I’m going show you something I came across and found interesting” or, “Watch this.”
- Show video/visual.
- Teacher asks, “What do you notice/wonder?” and “What are the first questions that come to mind?”
- Students share observations/questions with a partner first, then with the class (Think-Pair-Share). Students have ownership of the questions because they posed them.

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- Leave no student out of this questioning. Every student should have access to the scenario. No language or mathematical barriers. Low barrier to entry.
 - Teacher records questions (on chart paper or digitally-visible to class) and ranks them by popularity.
 - Determine which question(s) will be immediately pursued by the class. If you have a particular question in mind, and it isn't posed by students, you may have to do some skillful prompting to orient their question to serve the mathematical end. However, a good video should naturally lead to the question you hope they'll ask. You may wish to pilot your video on colleagues before showing it to students. If they don't ask the question you are after, your video may need some work.
 - Teacher asks for estimated answers in response to the question(s). Ask first for best estimates, then request estimates which are too high and too low. Students are not defining and defending parameters for making sense of forthcoming answers.
 - Teacher asks students to record their actual estimation for future reference.

Act 2 (Information Gathering):

Students gather information, draw on mathematical knowledge, understanding, and resources to answer the big question(s) from Act-1:

- Teacher asks, "What information do you need to answer our *main question*?"
- Students think of the important information they will need to answer their questions.
- Ask, "What mathematical tools do you have already at your disposal which would be useful in answering this question?"
- What mathematical tools might be useful which students don't already have? Help them develop those.
- Teacher offers smaller examples and asks probing questions.
 - What are you doing?
 - Why are you doing that?
 - What would happen if...?
 - Are you sure? How do you know?

Act 3 (The Reveal):

The payoff.

- Teacher shows the answer and validates students' solutions/answer.
- Teacher revisits estimates and determines closest estimate.
- Teacher compares techniques, and allows students to determine which is most efficient.

The Sequel:

- Students/teacher generalize the math to any case, and "algebrafy" the problem.
- Teacher poses an extension problem- best chance of student engagement if this extension connects to one of the many questions posed by students which were not the focus of Act 2, or is related to class discussion generated during Act 2.
- Teacher revisits or reintroduces student questions that were not addressed in Act 2.

Why Use 3-Act Tasks? A Teacher's Response

The short answer: It's what's best for kids!

If you want more, read on:

The need for students to make sense of problems can be addressed through tasks like these. The challenge for teachers is, to quote [Dan Meyer](#), “be less helpful.” (To clarify, being less helpful means to first allow students to generate questions they have about the picture or video they see in the first act, then give them information as they ask for it in act 2.) Less helpful does not mean give these tasks to students blindly, without support of any kind!

This entire process will likely cause some anxiety (for all). When jumping into 3-Act tasks for the first (second, third, . . .) time, students may not generate the suggested question. As a matter of fact, in this task about [proportions and scale](#), students may ask many questions that are curious questions, but have nothing to do with the mathematics you want them to investigate. One question might be “How is that ball moving by itself?” It’s important to record these and all other questions generated by students. This validates students' ideas. Over time, students will become accustomed to the routine of 3-act tasks and come to appreciate that there are certain kinds of mathematically answerable questions – most often related to quantity or measurement.

These kinds of tasks take time, practice and patience. When presented with options to use problems like this with students, the easy thing for teachers to do is to set them aside for any number of “reasons.” I've highlighted a few common “reasons” below with my commentary (in blue):

- This will take too long. I have a lot of content to cover. (Teaching students to think and reason is embedded in mathematical content at all levels - how can you **not** take this time)
- They need to be taught the skills first, then maybe I'll try it. (An important part of learning mathematics lies in productive struggle and learning to persevere [SMP 1]. What better way to discern what students know and are able to do than with a mathematical context [problem] that lets them show you, based on the knowledge they already have - prior to any new information. To quote John Van de Walle, “Believe in kids and they will, flat out, amaze you!”)
- My students can't do this. (Remember, whether you think they can or they can't, you're right!) (Also, this expectation of students persevering and solving problems is in every state's standards - and was there even before common core!)
- I'm giving up some control. (Yes, and this is a bit scary. You're empowering students to think and take charge of their learning. So, what can you do to make this *less* scary? **Do what we expect students to do:**
 - Persevere. Keep trying these and other open-beginning, -middle, and -ended problems. Take note of what's working and focus on it!
 - Talk with a colleague (work with a partner). Find that critical friend at school, another school, online. . .
 - Question (use #MTBoS on Twitter, or blogs, or Google: 3-act tasks).

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The benefits of students learning to question, persevere, problem solve, and reason mathematically far outweigh any of the reasons (read excuses) above. The time spent up front, teaching through tasks such as these and other open problems, creates a huge pay-off later on. However, it is important to note, that the problems themselves are worth nothing without teachers setting the expectation that students: question, persevere, problem solve, and reason mathematically on a daily basis. Expecting these from students, and facilitating the training of how to do this consistently and with fidelity is principal to success for both students and teachers.

Yes, all of this takes time. For most of my classes, mid to late September (we start school at the beginning of August) is when students start to become comfortable with what problem solving really is. It's not word problems - mostly. It's not the problem set you do after the skill practice in the textbook. Problem solving is what you do when you don't know what to do! This is difficult to teach kids and it does take time. But it is worth it! More on this in a future blog!

Tips:

One strategy I've found that really helps students generate questions is to allow them to talk to their peers about what they notice and wonder first (Act 1). Students of all ages will be more likely to share once they have shared and tested their ideas with their peers. This does take time. As you do more of these types of problems, students will become familiar with the format and their comfort level may allow you to cut the amount of peer sharing time down before group sharing.

What do you do if they don't generate the question suggested? Well, there are several ways that this can be handled. If students generate a similar question, use it. Allowing students to struggle through their question and ask for information is one of the big ideas here. Sometimes, students realize that they may need to solve a different problem before they can actually find what they want. If students are way off, in their questions, teachers can direct students, carefully, by saying something like: "You all have generated some interesting questions. I'm not sure how many we can answer in this class. Do you think there's a question we could find that would allow us to use our knowledge of mathematics to find the answer to (insert quantity or measurement)?" Or, if they are really struggling, you can, again carefully, say "You know, I gave this problem to a class last year (or class, period, etc) and they asked (insert something similar to the suggested question here). What do you think about that?" Be sure to allow students to share their thoughts.

After solving the main question, if there are other questions that have been generated by students, it's important to allow students to investigate these as well. Investigating these additional questions validates students' ideas and questions and builds a trusting, collaborative learning relationship between students and the teacher.

Overall, we're trying to help our students mathematize their world. We're best able to do that when we use situations that are relevant (no dog bandanas, please), engaging (create an intellectual need to know), and perplexing. If we continue to use textbook type problems that are too helpful, uninteresting, and let's face it, perplexing in all the wrong ways, we're not doing what's best for kids; we're training them to not be curious, not think, and worst of all . . . dislike math.

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3-Act Task Resources:

- www.estimated180.com
- www.visualpatterns.org
- [101 Questions](#)
- [Dan Meyer's 3-Act Tasks](#)
- [3-Act Tasks for Elementary and Middle School](#)
- [Andrew Stadel](#)
- [Jenise Sexton](#)
- [Graham Fletcher](#)
- [Fawn Nguyen](#)
- [Robert Kaplinsky](#)
- [Open Middle](#)
- Check out the Math Twitter Blog-o-Sphere (MTBoS) - you'll find tons of support and ideas!

Assessment Resources and Instructional Support Resources

The resource sites listed below are provided by the GADOE and are designed to support the instructional and assessment needs of teachers. All BLUE links will direct teachers to the site mentioned.

- Georgiastandards.org provides a gateway to a wealth of instructional links and information. Open the ELA/Math tab at the top to access specific math resources for GSE.
- Mathematics Georgia Standards of Excellence (MGSE) Frameworks are "models of instruction" designed to support teachers in the implementation of the GSE. The Georgia Department of Education, Office of Standards, Instruction, and Assessment has provided an example of the Curriculum Map for each grade level and examples of Frameworks aligned with the GSE to illustrate what can be implemented within the grade level. School systems and teachers are free to use these models as is; modify them to better serve classroom needs; or create their own curriculum maps, units and tasks. <http://bit.ly/1AJddmx>
- [The Teacher Resource Link](#) (TRL) is an application that delivers vetted and aligned digital resources to Georgia's teachers. TRL is accessible via the GaDOE "tunnel" in conjunction with SLDS using the single sign-on process. The content is aligned to Georgia Standards of Excellence and National Education Technology Standards and is pushed to teachers based on course schedule.
- **The Georgia Online Formative Assessment Resource (GOFAR) accessible through SLDS contains test items related to content areas assessed by the Georgia Milestones Assessment System and NAEP. Teachers and administrators can utilize the GOFAR to develop formative and summative assessments, aligned to the state-adopted content standards, to assist in informing daily instruction.**

The Georgia Online Formative Assessment Resource (GOFAR) provides the ability for Districts and Schools to assign benchmark and formative test items/tests to students in order to obtain information about student progress and instructional practice. GOFAR allows educators and their students to have access to a variety of test items – selected response and constructed response – that are aligned to the State-adopted content standards for Georgia's elementary, middle, and high schools.

Students, staff, and classes are repopulated and maintained through the State Longitudinal Data System (SLDS). Teachers and Administrators may view Exemplars and Rubrics in Item Preview. A scoring code may be distributed at a local level to help score constructed response items.

For GOFAR user guides and overview, please visit:

<https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx>

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- [Georgia Virtual School](#) content available on our Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.
- [Course/Grade Level WIKI](#) spaces are available to post questions about a unit, a standard, the course, or any other GSE math related concern. Shared resources and information are also available at the site.
- [Georgia Milestones Resources](#) are available to provide more information for the Georgia Milestones as provided by the GaDOE.
- **Georgia Milestones Assessment System_resources can be found at:**
<http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Milestones-Assessment-System.aspx>

Features of the Georgia Milestones Assessment System include:

- **Open-ended (constructed-response) items**
- **Norm-referenced items to complement the criterion-referenced information and to provide a national comparison;**
- **Transition to online administration over time, with online administration considered the primary mode of administration and paper-pencil as back-up until the transition is complete.**

Internet Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

GENERAL RESOURCES

[Illustrative Mathematics](#)

Standards are illustrated with instructional and assessment tasks, lesson plans, and other curriculum resources.

[Mathematics in Movies](#)

Short movie clips related to a variety of math topics.

[Mathematical Fiction](#)

Plays, short stories, comic books and novels dealing with math.

[The Shodor Educational Foundation](#)

This website has extensive notes, lesson plans and applets aligned with the standards.

[NEA Portal Arkansas Video Lessons on-line](#)

The NEA portal has short videos aligned to each standard. This resource may be very helpful for students who need review at home.

[Learnzillion](#)

This is another good resource for parents and students who need a refresher on topics.

[Math Words](#)

This is a good reference for math terms.

[National Library of Virtual Manipulatives](#)

Java must be enabled for this applet to run. This website has a wealth of virtual manipulatives helpful for use in presentation. The resources are listed by domain.

[Geogebra Download](#)

Free software similar to Geometer's Sketchpad. This program has applications for algebra, geometry, and statistics.

[Utah Resources](#)

Open resource created by the Utah Education Network.

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RESOURCES FOR PROBLEM-BASED LEARNING

[Dan Meyer's Website](#)

Dan Meyer has created many problem-based learning tasks. The tasks have great hooks for the students and are aligned to the standards in this [spreadsheet](#).

[Andrew Stadel](#)

Andrew Stadel has created many problem-based learning tasks using the same format as Dan Meyer.

[Robert Kaplinsky](#)

Robert Kaplinsky has created many tasks that engage students with real life situations.

[Geoff Krall's Emergent Math](#)

Geoff Krall has created a curriculum map structured around problem-based learning tasks.

ADDITIONAL RESOURCES

Burns, Marilyn. *About Mathematics*. (1992) Math Solutions Publications. Sausalito, California.

Burns, Marilyn (2007). *About Teaching Mathematics: A K-8 Resource*. Sausalito, CA: Scholastic, Inc.

Chapin, Suzanne and Johnson, Art (2006). *Math matters: Understanding the math you teach* 2nd Edition. Sausalito, CA: Math Solutions Publications

Katie Hendrickson, Illuminations Summer Institute., (2012) National Council of Teachers of Mathematics, Reston, Virginia.

National Research Council (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Swetz, Frank and J.S. Hartzler. *Mathematical Modeling*. (1991) National Council of Teachers of Mathematics. Reston, Virginia.

Van De Walle, John, *Elementary and Middle School Mathematics, Teaching Developmentally*, (2005).

Van de Walle, John A., Karp, Karen, Bay-Williams, Jennifer (2010). *Elementary and middle school mathematics: teaching developmentally* 7th edition. Boston: Allyn & Bacon (Pearson)