Unit Two Information

EOCT Domain & Weight: Equations - 30%

Curriculum Map: Reasoning with Linear Equations & Inequalities

Content Descriptors:
Concept 1: Create equations that describe numbers or relationships.
Concept 2: Understand solving equations as a process of reasoning & explain the reasoning. Solve equations and inequalities in one variable.
Concept 3: Solve systems of equations
Concept 4: Represent & solve equations & inequalities graphically.
Concept 5: Build a function that models a relationship between two quantities.
Concept 6: Understand the concept of a function and use function notation.
Concept 7: Interpret functions that arise in applications in terms of context.
Concept 8: Analyze functions using different representations.

Content from Frameworks:
Reasoning with Linear Equations & Inequalities

Unit Length: Approximately 36 days

Georgia Milestones Study Guide for Unit 2
TCSS – GSE Algebra 1 – Unit 2
Curriculum Map

Unit Rational:
Building on standards from middle school, students will analyze linear functions only. Students will:

- Investigate key features of graphs
- Create, solve, and model graphically linear equations and inequalities in one and two variables
- Create, solve and model graphically systems of linear equations in two variables
- Create and interpret systems of inequalities where applicable; for example, students will create a system to define the domain of a particular situation, such as situation limited to the first quadrant; the focus is not on solving systems of inequalities
- Rearrange formulas to highlight a quantity of interest
- Recognize arithmetic sequences as linear functions

Some of the Unit 2 standards will be repeated in Units 3, 4, and 5 as they also apply to quadratic and exponential functions.

Prerequisites: As identified by the GSE Frameworks

- Using the Pythagorean Theorem
- Understanding slope as a rate of change of one quantity in relation to another quantity
- Interpreting a graph
- Creating a table of values
- Working with functions
- Writing a linear equation
- Using inverse operations to isolate variables and solve equations
- Maintaining order of operations
- Understanding notation for inequalities
- Being able to read and write inequality symbols
- Graphing equations and inequalities on the coordinate plane
- Understanding and use properties of exponents
- Graphing points
- Choosing appropriate scales and label a graph

Length of Unit

<table>
<thead>
<tr>
<th>Concept 1</th>
<th>Concept 2</th>
<th>Concept 3</th>
<th>Concept 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create equations that describe numbers or relationships.</td>
<td>Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable.</td>
<td>Solve systems of equations</td>
<td>Represent and solve equations and inequalities graphically.</td>
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</table>

GSE Standards

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<tr>
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<tbody>
<tr>
<td>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational</td>
<td>Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify</td>
<td>Show and explain why the elimination method works to solve a system of two-variable equations.</td>
<td>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.</td>
</tr>
</tbody>
</table>
and exponential functions (integer inputs only).

**MGSE9-12.A.CED.2**
Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which has multiple variables.)

**MGSE9-12.A.CED.3**
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

**MGSE9-12.A.CED.4**
Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm’s law \( V = IR \) to highlight resistance \( R \); Rearrange area of a circle formula \( A = \pi r^2 \) to highlight the radius \( r \).

their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

**MGSE9-12.A.REI.3**
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (For example, given \( ax + 3 = 7 \), solve for \( x \)).

**MGSE9-12.A.REI.4**
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**MGSE9-12.A.REI.11**
Using graphs, tables, or successive approximations, show that the solution to the equation \( f(x) = g(x) \) is the \( x \)-value where the \( y \)-values of \( f(x) \) and \( g(x) \) are the same.

**MGSE9-12.A.REI.12**
Graph the solution set to a linear inequality in two variables.
## Lesson Essential Questions

<table>
<thead>
<tr>
<th>TCSS – GSE Algebra 1 – Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do I create linear equations from graphs?</td>
</tr>
<tr>
<td>How do I represent constraints by equations or inequalities?</td>
</tr>
<tr>
<td>How do I justify the solution to an equation?</td>
</tr>
<tr>
<td>How do I solve an equation in one variable?</td>
</tr>
<tr>
<td>How do I solve an inequality in one variable?</td>
</tr>
<tr>
<td>How do I prove that a system of two equations in two variables can be solved by multiplying and adding to produce a system with the same solutions?</td>
</tr>
<tr>
<td>How do I solve a system of linear equations graphically?</td>
</tr>
</tbody>
</table>

### Vocabulary

- **Algebra**
- **Coefficient**
- **Constant**
- **Constraints**
- **Coordinate Axes**
- **Equation**
- **Equivalent Expression**
- **Expression**
- **Factor**
- **Function**
- **Inequality**
- **Linear**
- **Quantity**
- **Simplify**
- **Solutions**
- **Term**
- **Variable**

- **Distributive Property**
- **Substitution**
- **Infinitely many**
- **Inequality**
- **Less than**
- **Less than or equal to**
- **No solution**
- **One solution**

- **Greater than**
- **Greater than or equal to**
- **Coordinate Plane**
- **Elimination Method Linear inequality**
- **Ordered Pair**
- **System of equations**
- **System of linear inequalities**

- **Accuracy**
- **Coordinate Plane**
- **Equation**
- **Variables**
- **Solutions**
- **Linear function**
- **Coordinates**
- **Intersect**

### Sample Assessment Items

**MGSE-9-12.A.REI.1**

Given that the following trapezoid has an area of 54 cm², what is the length of the unknown base?

Area of Trapezoid = \( \frac{1}{2} \cdot (\text{base}_1 + \text{base}_2) \cdot \text{height} \)

- **A.** 1 cm
- **B.** 5 cm
- **C.** 8 cm
- **D.** 16 cm

**MGSE9-12.A.REI.1**

Which of the following operations will solve Ohm’s law, \( V = IR \), for \( I \)?

- A. Subtract \( R \) from both sides.
- B. **Divide both sides by \( R \).**
- C. Subtract \( V \) from both sides.
- D. Divide both sides by \( I \).

**MGSE9-12.A.REI.5**

What is the solution for the system of equations represented by:

\[
4x - 2y = 12 \quad \text{and} \quad x = \frac{1}{2}y + 3
\]

- **A.** (4,2)
- **B.** (1, ½)
- **C.** **Infinitely many**
- **D.** No solution

**MGSE9-12.A.REI.10**

Which of the following is NOT a solution of the equation represented by the graph?

A. (4,2)
B. (1, ½)
C. **Infinitely many**
D. No solution
Rita reads a book at a steady pace. Rita graphs her progress through the book by putting the time in hours \( t \) on the horizontal axis and chapters remaining \( C \) on the vertical axis. Which equation describes Rita’s graph?

a. \( C = -3t + 24 \)
b. \( C = 3t - 24 \)
c. \( C = -24t + 3 \)
d. \( C = 24t + 3 \)

Publix sells chicken for $2.49/lb and pork for $3.19/lb. Mary buys \( c \) pounds of chicken and \( p \) pounds of pork. Which of the following inequalities represents that Mary only has $40 to spend?

a. \( 2.49c \leq 40 \)
b. \( 3.19p \leq 40 \)
c. \( c + p \leq 40 \)
d. \( 2.49c + 3.19p \leq 40 \)

Which equation shows \( 6(x + 4) = 2(y + 5) \) solved for \( y \)?

A. \( y = x + 3 \)
B. \( y = x + 5 \)
C. \( y = 3x + 7 \)
D. \( y = 3x + 17 \)

Which of the following systems of equations has a solution in which the \( x \)-value is greater than the \( y \)-value?

\[
\begin{align*}
5x - 2y &= 12 \\
-10x + 4y &= -20
\end{align*}
\]

a. \( (0, 2) \)
b. \( (1, 0) \)
c. \( (3, -4) \)
d. \( (4, -1) \)

Estimate the solution of the equation \( a(x) = b(x) \)?

\[ (9, 5.5) \]
## TCSS – GSE Algebra 1 – Unit 2

MGSE9-12.A.REI.12

Graph the following.

\[
\begin{align*}
\{ & \\
(Students should do this on graph paper – not M/C) \quad \text{Answer}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Resources – Concept 1</th>
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<th>Resources – Concept 4</th>
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<tr>
<td>❖ Foldable Graphic Organizer – Cumulative</td>
<td>❖ Graphic Organizer – Properties of Real Number</td>
<td>❖ Graphic Organizer Methods for Solving Systems of Equations</td>
<td>❖ Graphic Organizer –graphing transformations (limit to linear)</td>
</tr>
<tr>
<td>❖ Solving Literal Equations using variables (A.CED.4) – Power Point</td>
<td>❖ These tasks were taken from the GSE Frameworks.</td>
<td>❖ Tic Tac Toe</td>
<td>❖ Tic Tac Toe Activity (A.REI.11)</td>
</tr>
<tr>
<td>❖ Video for solving literal equations</td>
<td>❖ Selected terms &amp; symbols (notation is important)</td>
<td>❖ These tasks were taken from the GSE Frameworks.</td>
<td>❖ Kimi &amp; Jordan activator/discussion</td>
</tr>
<tr>
<td>❖ These tasks were taken from the CCGPS Frameworks.</td>
<td>❖ Jaden’s Phone Plan (A.REI.3)</td>
<td>❖ Systems of Weights - Act 1 only (A.REI.5) activator</td>
<td>❖ These tasks were taken from the GSE Frameworks.</td>
</tr>
<tr>
<td>❖ Lucy’s Linear Equations &amp; Inequalities (A.SSE.1, A.CED.1)</td>
<td>❖ Textbook Resources</td>
<td>❖ Solving Linear Equations FAL (A.REI.6)</td>
<td>❖ Talk is Cheap - great discussion and critical thinking task (A.REI.10&amp;11)</td>
</tr>
<tr>
<td>❖ Forget the Formula (A.CED.2,3,4) - extend activity</td>
<td>❖ Holt McDougal – Explorations in Core Math (A.REI.1)</td>
<td>❖ Good discussion</td>
<td>❖ Detention Hall Buy Out Spotlight task with video (A.REI.11)</td>
</tr>
<tr>
<td>❖ M&amp;M’s (A.CED.1,2 &amp; A.REI.1) - activator</td>
<td>❖ Textbook Resources</td>
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Holt McDougal – Explorations in Core Math p103-124 (A.REI.5)

TCSS

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## TCSS – GSE Algebra 1 – Unit 2

### Textbook Resources
- Holt McDougal – Explorations in Core Math p67-78 *(A.CED.1)*
- P41-42 Performance Task

### Differentiated Activities

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<tr>
<td>Solving word problems - worksheet <em>(A.CED.1)</em></td>
<td>Sorting Equations &amp; Identities FAL</td>
<td>Cube activity <code>cube1</code> <code>cube2</code> <code>cube1a</code> <code>cube2a</code></td>
<td>Linear Inequalities</td>
</tr>
<tr>
<td>FAL - The Largest Loser pages 73-87 <em>(N.Q.1,2,3 &amp; A.CED.2)</em></td>
<td>Reasoning about Equation Solving</td>
<td>Scavenger Hunt</td>
<td>Extension National Treasure activity [key]</td>
</tr>
<tr>
<td>Equation Solving</td>
<td><code>A.REI.3Practice</code></td>
<td></td>
<td>Guided Practice</td>
</tr>
</tbody>
</table>

### Resources recommended for Math Support

<p>| Interactive Vocabulary Site (differentiate how vocabulary is presented) | Online Practice | Systems with G.O. | Graphing Lines Simulator |
| These tasks were taken from the GSE Frameworks. | Guided notes or discussion | | These tasks were taken from the GSE Frameworks. |
| *Gas Pump <em>(A.REI.1,3 A.CED.1)</em> | <em>Inequality Practice Puzzles</em> | | <em>Rabbit Food Task pg89 (A.REI.12)</em> |
| *Bottomless Coffee Mug <em>(A.REI.1,3 A.CED.1)</em> | | | |</p>
<table>
<thead>
<tr>
<th>Concept 5</th>
<th>Concept 6</th>
<th>Concept 7</th>
<th>Concept 8</th>
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<td>Build a function that models a relationship between two quantities</td>
<td>Understand the concept of a function and use function notation</td>
<td>Interpret functions that arise in applications in terms of the context</td>
<td>Analyze functions using different representations</td>
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<tr>
<td>Write a function that describes a relationship between two quantities.</td>
<td>Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).</td>
<td>Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
<td>Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.</td>
</tr>
<tr>
<td>MGSE9-12.F.BF.1a</td>
<td>MGSE9-12.F.IF.2</td>
<td>MGSE9-12.F.IF.5</td>
<td>MGSE9-12.F.IF.7a</td>
</tr>
<tr>
<td>Determine an explicit expression and the recursive process (steps for calculation) from context.</td>
<td>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td>
<td>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</td>
<td>Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).</td>
</tr>
<tr>
<td>Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.</td>
<td>Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence a1=7, an=an-1 +2; the sequence sn = 2(n-1) + 7; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.</td>
<td>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.</td>
</tr>
</tbody>
</table>
**Lesson Essential Question**

<table>
<thead>
<tr>
<th>Why is the concept of a function important and how do I use function notation to show a variety of situations modeled by functions?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why are sequences functions?</td>
</tr>
<tr>
<td>How do I write recursive and explicit formulas for arithmetic sequences?</td>
</tr>
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</table>

**Lesson Essential Question**

<table>
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<tr>
<th>How do I use function notation to show a variety of situations modeled by functions?</th>
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<tbody>
<tr>
<td>How do I determine if the equation represents a function?</td>
</tr>
<tr>
<td>How do I model and interpret expressions for functions in terms of the situation they model?</td>
</tr>
<tr>
<td>What is a sequence and how can a sequence model be written as a function?</td>
</tr>
</tbody>
</table>

**Vocabulary**

- Linear Model
- Sequence
- Recursive
- Explicit
- Arithmetic sequence
- Output
- Input
- Estimate
- Average Rate of Change
- Constant Rate of Change
- Evaluate
- x-Intercept
- y-intercept
- Analyze
- Translate

**Sample Assessment Items**

**MGSE9-12.F.BF.1**

Katherine has $140 in the bank and is saving $6 per week. Abbie has $462 in the bank, but is spending at a rate of $10 per week. Which equation will determine when they will have the same amount of money in the bank?

a. $140 + 6x = 462 + 10x$
b. $140 + 6x = 462 - 10x$
c. $140 - 6x = 462 + 10x$
d. $140 + 10x = 462 - 6x$

**MGSE9-12.F.IF.1**

Which function is modeled in the table?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

**MGSE9-12.F.IF.4**

The graph can be described as:

![Graph](image)

**MGSE9-12.F.IF.7**

Sally decides to make and sell necklaces to earn money to buy a new computer. She plans to charge $5.25 per necklace.

a. Write a function that describes the revenue $R(n)$, in dollars, Sally will earn from selling $n$ necklaces.

\[ R(n) = 5.25n \]

b. What is a reasonable domain for this function?
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MGSE-12.F.BF.1a
A small swimming pool initially contains 400 gallons of water, and water is being added at a rate of 10 gallons per minute. Which expression represents the volume of the pool after \( t \) minutes?
- a. \(-10t + 400\)
- b. \(10t + 400\)
- c. \(400t + 10\)
- d. ....

MGSE-12.F.IF.2
If \( f(5) = 2(5) - 7 \), which function gives \( f(x) \)?
- a. \(f(x) = 2x\)
- b. \(f(x) = 5x\)
- c. \(f(x) = 2x - 7\)
- d. \(f(x) = 5x - 7\)

MGSE-12.F.IF.3
The first term in the sequence is -2.
Which function represents the sequence?
- a. \(a_n = a_{n-1} + 1\)
- b. \(a_n = a_{n-1} - 2\)
- c. \(a_n = a_{n-1} + 5\)
- d. \(a_n = a_{n-1} + 7\)

MGSE-12.F.IF.5
Turner Field, home of the Atlanta Braves, is capable of seating 56,790 fans. For each game, the amount of money that the Braves’ organization brings in as revenue is a function of the number of people, \( n \), in attendance. If each ticket costs $16, what is the domain of this function?
- a. \(0 \leq n \leq 56,790\)
- b. \(16 \leq n \leq 56,790\)
- c. \(0 \leq n \leq 908,640\)
- d. \(16 \leq n \leq 908,640\)

MGSE-12.F.IF.6
The rate of change is constant. Determine the rate of change and what the rate of change means for the situation.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>212</td>
</tr>
<tr>
<td>6</td>
<td>318</td>
</tr>
<tr>
<td>8</td>
<td>424</td>
</tr>
<tr>
<td>10</td>
<td>530</td>
</tr>
</tbody>
</table>

Since Sally is selling 1 necklace at a time and cannot sell negative necklaces, a reasonable domain for this function is the whole numbers.

c. Graph the function.
TCSS – GSE Algebra 1 – Unit 2

a. 1/53; your car travels 53 miles every hour
b. 10; your car travels for 10 hours
c. 53; your car travels 53 miles every hour
d. 212; your car travels 212 miles

a. x-intercept: 2; y-intercept: 2
b. x-intercept: 2; y-intercept: 4
c. x-intercept: 2; y-intercept: 4
d. x-intercept: 2; y-intercept: 4

MGSE9-12.F.IF.9
The table shows values for the function f(x), while the graph shows function g(x). Which function has the greater slope?

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>9</td>
<td>28</td>
</tr>
</tbody>
</table>

a. f(x)
b. g(x)
c. They are the same.
d. Not enough information.

Resources – Concept 5
- Instructional Strategies & Common Misconceptions (F.BF.1)
- Lake Algae activator/discussion
- Susita's Account

Resources – Concept 6
- Instructional Strategies & Common Misconceptions
- Is it a function? (F.IF.1)
- Find someone who (F.IF.2)
- Sequences Power Point Notes

Resources – Concept 7
- Instructional Strategies & Common Misconceptions
- Graphic Organizer ideas

Resources – Concept 8
- Rate of Change practice (F.IF.6)
- Guided notes on Average Rate of Change (F.IF.6)
<table>
<thead>
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<tbody>
<tr>
<td><strong>activator/discussion (F.BF.2)</strong> Exponential Growth word problems - practice worksheet</td>
</tr>
<tr>
<td><strong>Differentiated Activities</strong> Concept 5</td>
</tr>
<tr>
<td>These tasks were taken from the GSE Frameworks. Building and Combining Functions Task - do part or all (F.BF.2)</td>
</tr>
<tr>
<td><strong>Resources recommended for Math Support</strong></td>
</tr>
<tr>
<td>✓ Interactive Vocabulary Site (differentiate how vocabulary is presented) ✓ You’re Toast, Dude! Scaffolding Task – group activity</td>
</tr>
</tbody>
</table>
TCSS – GSE Algebra 1 – Unit 2

At the end of Unit 2 student’s should be able to say “I can…”

- Justify the solution of a linear equation and inequality in one variable.
- Justify the solution to a system of 2 equations in two variables.
- Solve a system of linear equations in 2 variables by graphing.
- Graph a linear inequality in 2 variables.
- Explain what it means when two graphs \( y = f(x) \) and \( y = g(x) \) intersect.
- Define and use function notation, evaluate functions at any point in the domain, give general statements about how \( f(x) \) behaves at different regions in the domain (as \( x \) gets very large or very negative, close to 0 etc.), and interpret statements that use function notation.
- Explain the difference and relationship between domain and range and find the domain and range of a function from a function equation, table or graph.
- Explain why sequences are functions.
- Interpret \( x \) and \( y \) intercepts, where the function is increasing or decreasing, where it is positive or negative, its end behaviors, given the graph, table or algebraic representation of a linear function in terms of the context of the function.
- Find and/or interpret appropriate domains and ranges for authentic linear functions.
- Calculate and interpret the average rate of change over a given interval of a function from a function equation, graph or table, and explain what that means in terms of the context of the function.
- Estimate the rate of change of a function from its graph at any point in its domain.
- Explain the relationship between the domain of a function and its graph in general and/or to the context of the function.
- Accurately graph a linear function by hand by identifying key features of the function such as the \( x \)- and \( y \)-intercepts and slope.
- Write recursive and explicit formulas for arithmetic sequences.