Models for Teaching Operations of Integers

These models have been adapted from [http://teachers.henrico.k12.va.us/math/hcpsalgebra1/](http://teachers.henrico.k12.va.us/math/hcpsalgebra1/).

The following are some everyday events that can be used to help students develop a conceptual understanding of addition and subtraction of integers.

- Getting rid of a negative is a positive. For example: Johnny used to cheat, fight and swear. Then he stopped cheating and fighting. Now he only has 1 negative trait so… (3 negative traits) - (2 negative traits) = (1 negative trait) or (-3) - (-2) = (-1)

- Using a credit card example can make this subtraction concept clearer. If you have spent money you don't have (-5) and paid off part of it (+3), you still have a negative balance (-2) as a debt, or (-5) + 3 = (-2).

- Draw a picture of a mountain, the shore (sea level) and the bottom of the ocean. Label sea level as 0.

Any of the following models can be used to help students understand the process of adding or subtracting integers. If students have trouble understanding and using one model you can show students how to use another model.

1. The Charged Particles Model (same as using two-color counters)

| When using charged particles to subtract, 3 – (-4) for example, you begin with a picture of 3 positive particles. | 📩ильно |
| Since there are no negative values to “take away”, you must use the Identity Property of Addition to rename positive 3 as 3 + 0. This is represented by 4 pairs of positive and negative particles that are equivalent to 4 zeros. | 📩ильно |
| Now that there are negative particles, you can “take away” 4 negative particles. | 📩ильно |
| The modeled problem shows that the result of subtracting 4 negative particles is actually like adding 4 positive particles. The result is 7 positive particles. This is a great way to show why 3 – (-4) = 3 + 4 = 7 | 📩ильно |
**Two-Color Counters Method**

When using two-colored counters you would use the yellow side to represent positive integers and the red side to represent negative numbers.

The problem represented is \(-3 - 5\).

<table>
<thead>
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<th>Counters Method</th>
<th>Diagram</th>
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<tbody>
<tr>
<td>When using two-colored counters you would use the yellow side to represent positive integers and the red side to represent negative numbers. The problem represented is (-3 - 5).</td>
<td>![Diagram of counters]</td>
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</tbody>
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\[ \begin{array}{c}
\text{Orange} + \text{Orange} \\
\text{Orange} + \text{Orange} \\
\text{Orange} + \text{Orange} \\
\text{Orange} + \text{Orange} \\
\end{array} \]
2. The Stack or Row Model

To model positive and negative integers, use colored linking cubes and graph paper. Graph paper and colored pencils will allow students to record problems and results. Students should also write the problems and answers numerically.

Create stacks or rows of numbers with the colored linking cubes and combine/compare the cubes. If the numbers have the same sign, then the cubes will be the same color. Stress that adding is like combining, so make a stack or row to show this.

\((-3) + (-4) = (-7)\)

If the numbers are not the same sign (color), for example \(-3 + 5\), you compare the stacks of different colors. Using the concept of zero pairs, the result is the difference between the stacks or the result is based on the higher stack. This is easy to see and understand.

For subtraction you create zeros by pairing one of each color. Then add as many zeros to the first number as needed so that you can take away what the problem calls for. Now physically take away the indicated amount and see what is left. The example problem shown is \(3 - (-4)\).

3. The Hot Air Balloon Model

Sand bags \(\text{(negative integers)}\) and Hot Air bags \(\text{(positive integers)}\) can be used to illustrate operations with integers. Bags can be put on (added to) the balloon or taken off (subtracted).

Here is an example: \(-3 - (-4) = ?\)
- The balloon starts at \(-3\) (think of the balloon being 3 feet below sea level or 3 feet below the level of a canyon) and you take off 4 sand bags.
- Now, think about what happens to a balloon if you remove sand bags, the balloon gets lighter. So, the balloon would go up 4 units.

If you think in terms of a vertical number line, it would start at \(-3\) and end up at 1, so \(-3 - (-4) = 1\). To help students make the connection between \(-3 - (-4)\) and \(-3 + (+4)\), present the addition and subtraction questions using the same numbers.

Another example would include the first addition question as \(9 + (-5)\) and the first subtraction question would then be \(9 - (+5)\). The students see that putting on 5 sand bags (negative) produces the same result as taking off 5 hot air bags (positive).
4. The Number Line Model

You can describe addition and subtraction of integers with a number line and a toy car. The car faces forward (to the right) to represent a positive direction. The car is moved forward to represent a positive integer. The car flips around backward (facing left) to represent a negative direction or subtraction. The car is moved backward (reverse) to represent a negative integer.

Example 1:  \( 4 + 4 = 8 \)

Example 2:  \( 4 + (-8) = -4 \)

Example 3:  \( 4 - (-4) = 8 \)
5. **Charged Particle Model for Multiplication**

The charged particle method can be used to illustrate multiplication of integers.

To begin, a model with a 0 charge is illustrated.

The 0 charge model will allow us to work with positive and negative integers.

**Example 1:** In this problem, \(3 \times (-2)\), three groups of two negative charges is added to the 0 charged field. The result is \((-6)\).

**Example 2:** \((-3) \times (-2) = ?\)