Unit Rate & Constant of Proportionality - 4 Lessons

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Topic B:

Unit Rate and Constant of Proportionality

7.RP.A.2b, 7.RP.A.2c, 7.RP.A.2d, 7.EE.B.4a

Focus Standard:

7.RP.A.2b Recognize and represent proportional relationships between quantities.
7.RP.A.2c b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2d c. Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t = pn$.
7.EE.B.4a d. Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Days: 4

Lesson 7: Unit Rate as the Constant of Proportionality (P)
Lessons 8–9: Representing Proportional Relationships with Equations (P)
Lesson 10: Interpreting Graphs of Proportional Relationships (P)

Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson
In Topic B, students learn to identify the constant of proportionality by finding the unit rate in the collection of equivalent ratios. They represent this relationship with equations of the form $y = kx$, where $k$ is the constant of proportionality (7.RP.A.2, 7.RP.A.2c). In Lessons 8 and 9, students derive the constant of proportionality from the description of a real-world context and relate the equation representing the relationship to a corresponding ratio table or graphical representation (7.RP.A.2b, 7.EE.B.4). Topic B concludes with students consolidating their graphical understandings of proportional relationships as they interpret the meanings of the points $(0,0)$ and $(1, r)$, where $r$ is the unit rate, in terms of the situation or context of a given problem (7.RP.A.2d).
Lesson 7: Unit Rate as the Constant of Proportionality

Student Outcomes

- Students identify the same value relating the measures of \( x \) and the measures of \( y \) in a proportional relationship as the constant of proportionality and recognize it as the unit rate in the context of a given situation.
- Students find and interpret the constant of proportionality within the contexts of problems.

Classwork

Example 1 (20 minutes): National Forest Deer Population in Danger?

Begin this lesson by presenting the example. Guide students to complete necessary information in the student materials.

**Example 1: National Forest Deer Population in Danger?**

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 216 deer in a 24 square mile plot of the forest. Do conservationists need to be worried?

a. Why does it matter if the deer population is not constant in a certain area of the National Forest?
   - Have students generate as many theories as possible (e.g., food supply, overpopulation, damage to land, etc.).

b. What is the population density of deer per square mile?
   - See table below.

Encourage students to make a chart to organize the data from the problem and then explicitly model finding the constant of proportionality. Students have already found unit rate in earlier lessons but have not identified it as the constant of proportionality.

- When we find the number of deer per 1 square mile, what is this called?
  - Unit rate
- When we look at the relationship between square miles and number of deer in the table below, how do we know if the relationship is proportional?
  - The square miles is always multiplied by the same value, 9 in this case.
Lesson 7
Unit Rate as the Constant of Proportionality

<table>
<thead>
<tr>
<th>Square Miles (x)</th>
<th>Number of Deer (y)</th>
<th>( \frac{144}{16} = 9 )</th>
<th>( \frac{117}{13} = 9 )</th>
<th>( \frac{216}{24} = 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>144</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>117</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>216</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- We call this constant (or same) value the “constant of proportionality.”
- So, the number of deer per square mile is 9, and the constant of proportionality is 9. Is that a coincidence, or will the unit rate of \( \frac{y}{x} \) and the constant of proportionality always be the same?

Allow for comments or observations but leave a lingering question for now.

- We could add the unit rate to the table so that we have 1 square mile in the first column and 9 in the second column. (Add this to the table for students to see). Does that help to guide your decision about the relationship between the unit rate of \( \frac{y}{x} \) and the constant of proportionality? We will see if your hypothesis remains true as we move through more examples.

The unit rate of deer per 1 square mile is \( \frac{9}{1} \).

Constant of Proportionality: \( k = 9 \)

Explain the meaning of the constant of proportionality in this problem: There are 9 deer for every 1 square mile of forest.

c. Use the unit rate of deer per square mile (or \( \frac{y}{x} \)) to determine how many deer are there for every 207 square miles.

\[
9(207) = 1,863 \text{ deer}
\]

d. Use the unit rate to determine the number of square miles in which you would find 486 deer?

\[
\frac{486}{9} = 54 \text{ square miles}
\]

Based upon the discussion of the questions above, answer the question: Do conservationists need to be worried? Be sure to support your answer with mathematical reasoning about rate and unit rate.

Review the vocabulary box with students.
Remind students that in the example with the deer population, we are looking for the number of deer per square mile, so the number of square miles could be defined as $x$, and the number of deer could be defined as $y$. The unit rate of deer per square mile is $\frac{144}{16}$, or 9. The constant of proportionality, $k$, is 9. The meaning in the context of Example 1 is as follows: There are 9 deer for every 1 square mile of forest.

Discussion

- How are the constant of proportionality and the unit rate of $\frac{y}{x}$ alike?
  - They both represent the value of the ratio of $y$ to $x$.

Example 2 (9 minutes): You Need WHAT???

While working on Example 2, encourage students to make a chart to organize the data from the problem.

Example 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needs 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needs the cookies the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets.

a. Is the number of cookies proportional to the number of cookie sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies baked.

<table>
<thead>
<tr>
<th>Number of Cookie Sheets</th>
<th>Number of Cookies Baked</th>
<th>$\frac{36}{2} = 18$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>72</td>
<td>$\frac{72}{4} = 18$</td>
</tr>
<tr>
<td>10</td>
<td>180</td>
<td>$\frac{180}{10} = 18$</td>
</tr>
<tr>
<td>16</td>
<td>288</td>
<td>$\frac{288}{16} = 18$</td>
</tr>
</tbody>
</table>

The unit rate of $\frac{y}{x}$ is **18**.

Constant of Proportionality: $k = 18$

Explain the meaning of the constant of proportionality in this problem: **There are 18 cookies per 1 cookie sheet.**
b. It takes 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 p.m., when will they finish baking the cookies?

- \[ \text{96 students (3 cookies each)} = 288 \text{ cookies} \]
- \[ \frac{288 \text{ cookies}}{18 \text{ cookies per sheet}} = 16 \text{ sheets of cookies} \]

If it takes 2 hours to bake 8 sheets, it will take 4 hours to bake 16 sheets of cookies. They will finish baking at 8:00 p.m.

**Example 3: (9 minutes) French Class Cooking**

Example 3: French Class Cooking

Suzette and Margo want to prepare crêpes for all of the students in their French class. A recipe makes 20 crêpes with a certain amount of flour, milk, and 2 eggs. The girls already know that they have plenty of flour and milk to make 50 crêpes, but they need to determine the number of eggs they will need for the recipe because they are not sure they have enough.

a. Considering the amount of eggs necessary to make the crêpes, what is the constant of proportionality?

\[ \frac{2 \text{ eggs}}{20 \text{ crêpes}} = \frac{1 \text{ egg}}{10 \text{ crêpes}}; \text{ the constant of proportionality is } \frac{1}{10}. \]

b. What does the constant or proportionality mean in the context of this problem?

One egg is needed to make 10 crêpes.

c. How many eggs are needed to make 50 crêpes?

\[ 50 \left( \frac{1}{10} \right) = 5; \text{ five eggs are needed to make 50 crêpes.} \]

**Closing (2 minutes)**

- What is another name for the constant that relates the measures of two quantities?
  - Another name for the constant is the constant of proportionality.
- How is the constant of proportionality related to the unit rate of \( \frac{y}{x} \)?
  - They represent the value of the ratio \( y : x \).

**Lesson Summary**

If a proportional relationship is described by the set of ordered pairs that satisfies the equation \( y = kx \), where \( k \) is a positive constant, then \( k \) is called the constant of proportionality.

**Exit Ticket (5 minutes)**
Lesson 7: Unit Rate as the Constant of Proportionality

Exit Ticket

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will determine the total cost of the sodas. Who is right and why?
Exit Ticket Sample Solutions

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will determine the total cost of the sodas. Who is right and why?

Susan is correct. The table below shows that if you multiply the unit price, say 0.50, by the number of people, say 12, you will determine the total cost of the soda. I created a table to model the proportional relationship. I used a unit price of 0.50 to make the comparison.

<table>
<thead>
<tr>
<th>Number of People</th>
<th>Total Cost of Soda (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td>2.00</td>
</tr>
<tr>
<td>12</td>
<td>6.00</td>
</tr>
</tbody>
</table>

I used the same values to compare to John. total cost 12 people = ?

The total cost is $6 and there 12 people. $6 12 = 1 2 which is 0.50 or the unit cost, not the total cost.

Problem Set Sample Solutions

For each of the following problems, define the constant of proportionality to answer the follow-up question.

1. Bananas are $0.59/pound.
   a. What is the constant of proportionality, k?
      
      \[ \text{The constant of proportionality (k) is 0.59.} \]
   b. How much will 25 pounds of bananas cost?
      \[ 25 \times 0.59 = 14.75 \]

2. The dry cleaning fee for 3 pairs of pants is $18.
   a. What is the constant of proportionality?
      \[ \frac{18}{3} = 6, \text{ so } k \text{ is } 6. \]
   b. How much will the dry cleaner charge for 11 pairs of pants?
      \[ 6 \times 11 = 66 \]

3. For every $5 that Micah saves, his parents give him $10.
   a. What is the constant of proportionality?
      \[ \frac{10}{5} = 2, \text{ so } k \text{ is } 2. \]
   b. If Micah saves $150, how much money will his parents give him?
      \[ 2 \times 150 = 300 \]
4. Each school year, the 7th graders who study Life Science participate in a special field trip to the city zoo. In 2010, the school paid $1,260 for 84 students to enter the zoo. In 2011, the school paid $1,050 for 70 students to enter the zoo. In 2012, the school paid $1,395 for 93 students to enter the zoo.

a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?

<table>
<thead>
<tr>
<th>Number of Students</th>
<th>Price</th>
<th>( \frac{1260}{84} = 15 )</th>
<th>( \frac{1050}{70} = 15 )</th>
<th>( \frac{1395}{93} = 15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
<td>1,260</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>1,050</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>1,395</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Explain why or why not.

The price is proportional to the number of students because the ratio of the entrance fee paid per student was the same.

\( \frac{1260}{84} = 15 \)

\( \frac{1050}{70} = 15 \)

\( \frac{1395}{93} = 15 \)

c. Identify the constant of proportionality and explain what it means in the context of this situation.

The constant of proportionality \( k \) is 15. This represents the price per student.

d. What would the school pay if 120 students entered the zoo?

\( 120(15) = \$1,800 \)

e. How many students would enter the zoo if the school paid \$1,425?

\( \frac{1425}{15} = 95 \) students
Lesson 7: Unit Rate as the Constant of Proportionality

Classwork

Example 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 216 deer in a 24 square mile plot of the forest. Do conservationists need to be worried?

a. Why does it matter if the deer population is not constant in a certain area of the National Forest?

b. What is the population density of deer per square mile?

The unit rate of deer per 1 square mile is ______.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

c. Use the unit rate of deer per square mile (or \( \frac{y}{x} \)) to determine how many deer are there for every 207 square miles.

d. Use the unit rate to determine the number of square miles in which you would find 486 deer?
Example 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needs 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needs the cookies the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets.

a. Is the number of cookies proportional to the number of cookie sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies baked.

Table:

<table>
<thead>
<tr>
<th>Number of Sheets</th>
<th>Number of Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>72</td>
</tr>
<tr>
<td>6</td>
<td>108</td>
</tr>
</tbody>
</table>

The unit rate of \( \frac{Y}{X} \) is \___________.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

b. It takes 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 p.m., when will they finish baking the cookies?
Example 3: French Class Cooking

Suzette and Margo want to prepare crêpes for all of the students in their French class. A recipe makes 20 crêpes with a certain amount of flour, milk, and 2 eggs. The girls already know that they have plenty of flour and milk to make 50 crêpes, but they need to determine the number of eggs they will need for the recipe because they are not sure they have enough.

a. Considering the amount of eggs necessary to make the crêpes, what is the constant of proportionality?

b. What does the constant or proportionality mean in the context of this problem?

c. How many eggs are needed to make 50 crêpes?
Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where $k$ is a positive constant, then $k$ is called the constant of proportionality.

Problem Set

For each of the following problems, define the constant of proportionality to answer the follow-up question.

1. Bananas are $0.59/pound.
   a. What is the constant of proportionality, $k$?
   b. How much will 25 pounds of bananas cost?

2. The dry cleaning fee for 3 pairs of pants is $18.
   a. What is the constant of proportionality?
   b. How much will the dry cleaner charge for 11 pairs of pants?

3. For every $5 that Micah saves, his parents give him $10.
   a. What is the constant of proportionality?
   b. If Micah saves $150, how much money will his parents give him?

4. Each school year, the 7th graders who study Life Science participate in a special field trip to the city zoo. In 2010, the school paid $1,260 for 84 students to enter the zoo. In 2011, the school paid $1,050 for 70 students to enter the zoo. In 2012, the school paid $1,395 for 93 students to enter the zoo.
   a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?
   b. Explain why or why not.
   c. Identify the constant of proportionality and explain what it means in the context of this situation.
   d. What would the school pay if 120 students entered the zoo?
   e. How many students would enter the zoo if the school paid $1,425?
Lesson 8: Representing Proportional Relationships with Equations

Student Outcomes

- Students use the constant of proportionality to represent proportional relationships by equations in real-world contexts as they relate the equations to a corresponding ratio table and/or graphical representation.

Classwork

Discussion (5 minutes)

**Points to remember:**

- Proportional relationships have a constant ratio, or unit rate.
- The constant ratio, or unit rate of \( \frac{y}{x} \), can also be called the constant of proportionality.

**Discussion Notes**

How could we use what we know about the constant of proportionality to write an equation?

Discuss important facts.

Encourage students to begin thinking about how we can model a proportional relationship using an equation by asking the following probing questions:

- If we know that the constant of proportionality, \( k \), is equal to \( \frac{y}{x} \) for a given set of ordered pairs, \( x \) and \( y \), then we can write \( k = \frac{y}{x} \). How else could we write this equation? What if we know the \( x \)-values and the constant of proportionality, but do not know the \( y \)-values? Could we rewrite this equation to solve for \( y \)?

Elicit ideas from students. Apply their ideas in the examples below. Provide the context of the examples below to encourage students to test their thinking.

Students should note the following in their student materials: \( k = \frac{y}{x} \) and eventually \( y = kx \) (You may need to add this second equation after Example 1).
Examples 1–2 (33 minutes)

Write an equation that will model the real-world situation.

Example 1: Do We Have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip, and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 222 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother’s Gas Record

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
</tbody>
</table>

a. Find the constant of proportionality and explain what it represents in this situation.

\[
\begin{array}{c|c}
\text{Gallons} & \text{Miles Driven} \\
8 & 224 \\
10 & 280 \\
4 & 112 \\
\end{array}
\]

\[
\frac{224}{8} = 28 \quad \text{or} \quad \frac{280}{10} = 28 \quad \text{or} \quad \frac{112}{4} = 28
\]

The constant of proportionality (k) is 28. The car travels 28 miles for every one gallon of gas.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

\[y = 28x \quad \text{or} \quad m = 28g\]

c. Knowing that there is a half-gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

No, she will not make it because she gets 28 miles to one gallon. Since she has \( \frac{1}{2} \) gallon remaining in the gas tank, she can travel 14 miles. Since the nearest gas station is 26 miles away, she will not have enough gas.

d. Using the equation found in part (b), determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways: once using the constant of proportionality and once using an equation.

\[
\text{Using arithmetic: } 28(18) = 504
\]

\[
\text{Using an equation: } m = 28g \quad \text{– Use substitution to replace the } g \text{ (gallons of gas) with } 18.
\]

\[
m = 28(18) \quad \text{– This is the same as multiplying by the constant of proportionality.}
\]

\[
m = 504
\]

Your mother can travel 504 miles on 18 gallons of gas.
e. Using the constant of proportionality, and then the equation found in part (b), determine how many gallons of gas would be needed to travel 750 miles.

Using arithmetic: \( \frac{750}{28} = 26.8 \)

Using algebra: \( \frac{m}{750} = \frac{28}{28} \)

\( m = 28g \)

\( \frac{1}{28} \cdot 750 = \frac{1}{28} \cdot 28g \)

\( 26.8 = 1g \)

- Use substitution to replace the \( m \) (miles driven) with 750.
- This equation demonstrates dividing by the constant of proportionality or using the multiplicative inverse to solve the equation.

26.8 (rounded to the nearest tenth) gallons would be needed to drive 750 miles.

Have students write the pairs of numbers in the chart as ordered pairs. Explain that in this example \( x \) represents the number of gallons and \( y \) represents the number of miles driven. Remind students to think of the constant of proportionality as \( k = \frac{y}{x} \). In this case, the constant of proportionality is a certain number of miles divided by a certain number of gallons of gas. This constant is the same as the unit rate of miles per gallon of gas. Remind students that you will use the constant of proportionality (or unit rate) as a multiplier in your equation.

- Write equation(s) that will relate the miles driven to the number of gallons of gas.

In order to write the equation to represent this situation, direct students to think of the independent and dependent variables that are implied in this problem.

- Which part depends on the other for its outcome?
  - The number of miles driven depends on the number of gallons of gas that are in the gas tank.
- Which is the dependent variable: the number of gallons of gas or the amount of miles driven?
  - The number of miles is the dependent variable, and the number of gallons is the independent variable.

Tell students that \( x \) is usually known as the independent variable, and \( y \) is known as the dependent variable.

Remind students that the constant of proportionality can also be expressed as \( \frac{y}{x} \) from an ordered pair. It is the value of the ratio of the dependent variable to the independent variable.

- When \( x \) and \( y \) are graphed on a coordinate plane, which axis would show the values of the dependent variable?
  - \( y \)-axis
- The independent variable?
  - \( x \)-axis

Tell students that any variable may be used to represent the situation as long as it is known that in showing a proportional relationship in an equation that the constant of proportionality is multiplied by the independent variable. In this problem, students can write \( y = 28x \), or \( m = 28g \). We are substituting \( k \) with 28 in the equation \( y = kx \), or \( m = kg \).

Tell students that this equation models the situation and will provide them with a way to determine either variable when the other is known. If the equation is written so a variable can be substituted with the known information, then students can use algebra to solve the equation.
Example 2: Andrea’s Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits) of tourists. People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours she needs to draw the portraits.

The constant of proportionality is $\frac{3}{2}$, which means that Andrea can draw 3 portraits in 2 hours, or can complete $1\frac{1}{2}$ portraits in 1 hour.

Tell students that these ordered pairs can be used to generate the constant of proportionality and write the equation for this situation. Remember that $y = \frac{v}{x}$.
Closing (2 minutes)

- How can unit rate be used to write an equation relating two variables that are proportional?
  - The unit rate of \( \frac{y}{x} \) is the constant of proportionality, \( k \). After computing the value for \( k \), it may be substituted in place of \( k \) in the equation \( y = kx \). The constant of proportionality can be multiplied by the independent variable to find the dependent variable, and the dependent variable can be divided by the constant of proportionality to find the independent variables.

Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation \( y = kx \), where \( k \) is a positive constant, then \( k \) is called the constant of proportionality. The constant of proportionality expresses the multiplicative relationship between each \( x \)-value and its corresponding \( y \)-value.

Exit Ticket (5 minutes)
Lesson 8: Representing Proportional Relationships with Equations

Exit Ticket

John and Amber work at an ice cream shop. The hours worked and wages earned are given for each person.

<table>
<thead>
<tr>
<th>John’s Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (in hours)</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

1. Determine if John’s wages are proportional to time. If they are, determine the unit rate of \( \frac{y}{x} \). If not, explain why they are not.

2. Determine if Amber’s wages are proportional to time. If they are, determine the unit rate of \( \frac{y}{x} \). If not, explain why they are not.
3. Write an equation for both John and Amber that models the relationship between their wage and the time they worked. Identify the constant of proportionality for each. Explain what it means in the context of the situation.

4. How much would each worker make after working 10 hours? Who will earn more money?

5. How long will it take each worker to earn $50?
Exit Ticket Sample Solutions

John and Amber work at an ice cream shop. The hours worked and wages earned are given for each person.

<table>
<thead>
<tr>
<th>John’s Wages</th>
<th>Amber’s Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (in hours)</td>
<td>Wages (in dollars)</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
</tr>
</tbody>
</table>

1. Determine if John’s wages are proportional to time. If they are, determine the unit rate of \( \frac{y}{x} \). If not, explain why they are not.

   Yes, the unit rate is 9. The collection of ratios is equivalent.

2. Determine if Amber’s wages are proportional to time. If they are, determine the unit rate of \( \frac{y}{x} \). If not, explain why they are not.

   Yes, the unit rate is 8. The collection of ratios is equivalent.

3. Write an equation for both John and Amber that models the relationship between their wage and the time they worked. Identify the constant of proportionality for each. Explain what it means in the context of the situation.

   John: \( w = 9h \); the constant of proportionality is 9; John earns $9 for every hour he works.

   Amber: \( w = 8h \); the constant of proportionality is 8; Amber earns $8 for every hour she works.

4. How much would each worker make after working 10 hours? Who will earn more money?

   After 10 hours John will earn $90 because 10 hours is the value of the independent variable which should be multiplied by \( k \), the constant of proportionality. \( w = 9h; w = 9(10); w = 90 \). After 10 hours, Amber will earn $80 because her equation is \( w = 8h; w = 8(10); w = 80 \). John will earn more money than Amber in the same amount of time.

5. How long will it take each worker to earn $50?

   To determine how long it will take John to earn $50, the dependent value will be divided by 9, the constant of proportionality. Algebraically, this can be shown as a one-step equation: \( 50 = 9h; \left( \frac{1}{9} \right) 50 = \left( \frac{1}{9} \right) 9h; \frac{50}{9} = 1h; 5.56 = h \) (round to the nearest hundredth). It will take John nearly 6 hours to earn $50. To find how long it will take Amber to earn $50, divide by 8, the constant of proportionality. \( 50 = 8h; \left( \frac{1}{8} \right) 50 = \left( \frac{1}{8} \right) 8h; \frac{50}{8} = 1h; 6.25 = h \). It will take Amber 6.25 hours to earn $50.
Problem Set Sample Solutions

Write an equation that will model the proportional relationship given in each real-world situation.

1. There are 3 cans that store 9 tennis balls. Consider the number of balls per can.
   a. Find the constant of proportionality for this situation.
      \[
      \frac{9 \text{ balls}}{3 \text{ cans}} = \frac{3 \text{ balls}}{1 \text{ can}}
      \]
      The constant of proportionality is 3.
   b. Write an equation to represent the relationship.
      \[B = 3C\]

2. In 25 minutes, Li can run 10 laps around the track. Determine the number of laps she can run per minute.
   a. Find the constant of proportionality in this situation.
      \[
      \frac{10 \text{ laps}}{25 \text{ minutes}} = \frac{2 \text{ laps}}{5 \text{ minute}}
      \]
      The constant of proportionality is \(\frac{2}{5}\).
   b. Write an equation to represent the relationship.
      \[L = \frac{2}{5}M\]

3. Jennifer is shopping with her mother. They pay $2 per pound for tomatoes at the vegetable stand.
   a. Find the constant of proportionality in this situation.
      \[
      \frac{2 \text{ dollars}}{1 \text{ pound}} = \frac{2 \text{ dollars}}{1 \text{ pound}}
      \]
      The constant of proportionality is 2.
   b. Write an equation to represent the relationship.
      \[D = 2P\]

4. It cost $15 to send 3 packages through a certain shipping company. Consider the number of packages per dollar.
   a. Find the constant of proportionality for this situation.
      \[
      \frac{3 \text{ packages}}{15 \text{ dollars}} = \frac{3 \text{ packages}}{15 \text{ dollar}}
      \]
      The constant of proportionality is \(\frac{1}{5}\).
   b. Write an equation to represent the relationship.
      \[P = \frac{1}{5}D\]
5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded onto personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $58.00 per month offered by another company. Which is the better buy?

<table>
<thead>
<tr>
<th>Number of Songs Purchased (S)</th>
<th>Total Cost (C)</th>
<th>Constant of Proportionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>36</td>
<td>( \frac{36}{40} = \frac{9}{10} = 0.9 )</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
<td>( \frac{18}{20} = \frac{9}{10} = 0.9 )</td>
</tr>
<tr>
<td>12</td>
<td>10.80</td>
<td>( \frac{10.80}{12} = \frac{9}{10} = 0.9 )</td>
</tr>
<tr>
<td>5</td>
<td>4.50</td>
<td>( \frac{4.50}{5} = \frac{9}{10} = 0.9 )</td>
</tr>
</tbody>
</table>

a. Find the constant of proportionality for this situation.

The constant of proportionality \( k \) is 0.9.

b. Write an equation to represent the relationship.

\( C = 0.9S \)

c. Use your equation to find the answer to Susan’s question above. Justify your answer with mathematical evidence and a written explanation.

Compare the flat fee of $58 per month to $0.90 per song. If \( C = 0.9S \) and we substitute \( S \) with 60 (the number of songs), then the result is \( C = 0.9(60) = 54 \). She would spend $54 on songs if she bought 60 songs. If she maintains the same number of songs, the charge of $0.90 per song would be cheaper than the flat fee of $58 per month.

6. Allison’s middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee, as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T’s and More charges $8 per shirt. Which company should they use?

<table>
<thead>
<tr>
<th>Number of Shirts (S)</th>
<th>Total Cost (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>375</td>
</tr>
<tr>
<td>75</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Print-o-Rama: $95 total cost

Value T’s and More: $8 per shirt
a. Does either pricing model represent a proportional relationship between the quantity of t-shirts and the total cost? Explain.

*The unit rate of \( \frac{y}{x} \) for Print-o-Rama is not constant. The graph for Value T’s and More is proportional since the ratios are equivalent (8) and the graph shows a line through the origin.*

b. Write an equation relating cost and shirts for Value T’s and More.

\[ C = 8S \text{ for Value T’s and More} \]

c. What is the constant of proportionality of Value T’s and More? What does it represent?

8; the cost of one shirt is $8.

d. How much is Print-o-Rama’s set-up fee?

The set-up fee is $25

e. Write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.

*Since we plan on a purchase of 90 shirts, we should choose Print-o-Rama.*

Print-o-Rama: \[ C = 7S + 25; C = 7(90) + 25; C = 655 \]

Value T’s and More: \[ C = 8S; C = 8(90); C = 720 \]
Lesson 8: Representing Proportional Relationships with Equations

Classwork
Points to remember:
- Proportional relationships have a constant ratio, or unit rate.
- The constant ratio, or unit rate of $\frac{y}{x}$, can also be called the constant of proportionality.

Discussion Notes
How could we use what we know about the constant of proportionality to write an equation?
Example 1: Do We have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother’s Gas Record

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
</tbody>
</table>

a. Find the constant of proportionality and explain what it represents in this situation.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

c. Knowing that there is a half-gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

d. Using the equation found in part (b), determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways: once using the constant of proportionality and once using an equation.

e. Using the constant of proportionality, and then using the equation found in part (b), determine how many gallons of gas would be needed to travel 750 miles.
Example 2: Andrea's Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits) of tourists. People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours she needs to draw the portraits.

![Graph showing the relationship between the number of portraits drawn and time spent drawing them.]

a. Write several ordered pairs from the graph and explain what each ordered pair means in the context of this graph.

b. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

c. Determine the constant of proportionality and explain what it means in this situation.
Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation \( y = kx \), where \( k \) is a positive constant, then \( k \) is called the constant of proportionality. The constant of proportionality expresses the multiplicative relationship between each \( x \)-value and its corresponding \( y \)-value.

Problem Set

Write an equation that will model the proportional relationship given in each real-world situation.

1. There are 3 cans that store 9 tennis balls. Consider the number of balls per can.
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.

2. In 25 minutes Li can run 10 laps around the track. Determine the number of laps she can run per minute.
   a. Find the constant of proportionality in this situation.
   b. Write an equation to represent the relationship.

3. Jennifer is shopping with her mother. They pay $2 per pound for tomatoes at the vegetable stand.
   a. Find the constant of proportionality in this situation.
   b. Write an equation to represent the relationship.

4. It costs $15 to send 3 packages through a certain shipping company. Consider the number of packages per dollar.
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.

5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded on to personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $58.00 per month offered by another company. Which is the better buy?
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.
   c. Use your equation to find the answer to Susan’s question above. Justify your answer with mathematical evidence and a written explanation.
6. Allison’s middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee, as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T’s and More charges $8 per shirt. Which company should they use?

<table>
<thead>
<tr>
<th># shirts</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>375</td>
</tr>
<tr>
<td>75</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

a. Does either pricing model represent a proportional relationship between the quantity of t-shirts and the total cost? Explain.
b. Write an equation relating cost and shirts for Value T’s and More.
c. What is the constant of proportionality Value T’s and More? What does it represent?
d. How much is Print-o-Rama’s set-up fee?
e. Write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.
Lesson 9: Representing Proportional Relationships with Equations

Student Outcomes

- Students use the constant of proportionality to represent proportional relationships by equations in real-world contexts as they relate the equations to a corresponding ratio table and/or graphical representation.

Classwork

Students will begin to write equations in two variables. They will analyze data that will help them understand the constant of proportionality and write the equation with two variables. The teacher may need to explicitly connect the graphical and tabular representations by modeling them side-by-side.

Example 1 (18 minutes): Jackson’s Birdhouses

Example 1: Jackson’s Birdhouses

Jackson and his grandfather constructed a model for a birdhouse. Many of their neighbors offered to buy the birdhouses. Jackson decided that building birdhouses could help him earn money for his summer camp, but he is not sure how long it will take him to finish all of the requests for birdhouses. If Jackson can build 7 birdhouses in 5 hours, write an equation that will allow Jackson to calculate the time it will take him to build any given number of birdhouses, assuming he works at a constant rate.

a. Write an equation that you could use to find out how long it will take him to build any number of birdhouses.

\[ H = \frac{5}{7} B \]

Define the variables. \( B \) represents the number of birdhouses and \( H \) represents the number of hours (time constructing birdhouses).

- Does it matter which of these variables is independent or dependent?
  - No. The number of birdhouses made could depend on how much time Jackson can work or the amount of time he works could depend on how many birdhouses he needs to make.

- If it is important to determine the number of birdhouses that can be built in one hour, what is the constant of proportionality?
  - \( \frac{number\ of\ birdhouses}{number\ of\ hours} = \frac{7}{5} \) or 1.4.

- What does that mean in the context of this situation?
  - It means that Jackson can build 1.4 birdhouses in one hour or one entire birdhouse and part of a second birdhouse in one hour.
• If it is important to determine the number of hours it takes to build one birdhouse, what is the constant of proportionality?

\[
\frac{\text{number of hours}}{\text{number of birdhouses}} = \frac{5}{7} \text{ or } 0.71, \text{ which means that it takes him } \frac{5}{7} \text{ of an hour to build one birdhouse or } \left(\frac{5}{7}\right)(60) = 43 \text{ minutes to build one birdhouse.}
\]

• This part of the problem asks you to write an equation that will let Jackson determine how long it will take him to build any number of birdhouses, so we want to know the value of \(H\). This forces \(H\) to be the dependent variable and \(B\) to be the independent variable. Our constant of proportionality will be \(\frac{H}{B}\) or \(\frac{y}{x}\), which is \(\frac{5}{7}\), so we will use the equation \(H = \frac{5}{7}B\).

Use the equation above to determine the following:

b. How many birdhouses can Jackson build in 40 hours?

\[
\text{If } H = B \text{ and } H = 40, \text{ then substitute } 40 \text{ in the equation for } H \text{ and solve for } B \text{ since the question asks for the number or birdhouses.}
\]

\[
40 = \left(\frac{5}{7}\right)B
\]

\[
\left(\frac{7}{5}\right)40 = \left(\frac{7}{5}\right)\left(\frac{5}{7}\right)B
\]

\[
56 = B
\]

Jackson can build 56 birdhouses in 40 hours.

c. How long will it take Jackson to build 35 birdhouses? Use the equation from part (a) to solve the problem.

\[
\text{If } H = \frac{5}{7}B \text{ and } B = 35, \text{ then substitute } 35 \text{ into the equation for } B; H = \left(\frac{5}{7}\right)(35); H = 25. \text{ It will take Jackson 25 hours to build 35 birdhouses.}
\]

d. How long will it take to build 71 birdhouses? Use the equation from part (a) to solve the problem.

\[
\text{If } H = \frac{5}{7}B \text{ and } B = 71, \text{ then substitute } 71 \text{ for } B \text{ into the equation; } H = \left(\frac{5}{7}\right)(71); H = 50.7 \text{ (rounded to the nearest tenth). It will take Jeff 50 hours and 42 minutes } (60 \text{ (0.7)}) \text{ to build 71 birdhouses.}
\]

Remind students that while one may work for a fractional part of an hour, a customer will not want to buy a partially built birdhouse. Tell students that some numbers can be left as non-integer answers (e.g., parts of an hour that can be written as minutes), but others must be rounded to whole numbers (e.g., the number of birdhouses completed or sold). All of this depends on the context. We must consider the real-life context before we determine if and how we round.

**Example 2 (17 minutes): Al’s Produce Stand**

Let students select any two pairs of numbers from either Al’s Produce Stand or Barbara’s Produce Stand to calculate the constant of proportionality (\(k = \text{dependent/independent}\)). In order to determine the unit price, students need to divide the cost (dependent variable) by the number of ears of corn (independent variable). Lead them through the following questions to organize their thinking.
### Lesson 9: Representing Proportional Relationships with Equations

- **Which makes more sense:** to use a unit rate of “ears of corn per dollar” or of “dollars (or cents) per ear of corn”?
  - The cost per ear of corn makes more sense because corn is sold as an entire ear of corn, not part of an ear of corn.

- Based on the previous question, which will be the independent variable?
  - The independent variable will be the number of ears of corn.

- Which will be the dependent variable and why?
  - The cost will be the dependent variable because the cost depends on the number of ears of corn purchased.

Have students volunteer to share the pair of numbers they used to determine the unit rate, or constant of proportionality, and compare the values for Al’s Produce Stand and for Barbara’s Produce Stand.

- **Al’s Produce Stand:** 0.25 and **Barbara’s Produce Stand:** 0.24

- How do you write an equation for a proportional relationship?
  - \( y = kx \)

- Write the equation for Al’s Produce Stand:
  - \( y = 0.25x \)

- Write the equation for Barbara’s Produce Stand:
  - \( y = 0.24x \)

---

**Example 2: Al’s Produce Stand**

Al’s Produce Stand sells 6 ears of corn for $1.50. Barbara’s Produce Stand sells 13 ears of corn for $3.12. Write two equations, one for each produce stand, that models the relationship between the number of ears of corn sold and the cost. Then use each equation to help complete the tables below.

**Al’s Produce Stand:** \( y = 0.25x \); where \( x \) represents the number of ears of corn and \( y \) represents the cost

**Barbara’s Produce Stand:** \( y = 0.24x \); where \( x \) represents the number of ears of corn and \( y \) represents the cost

<table>
<thead>
<tr>
<th>Ears</th>
<th>6</th>
<th>14</th>
<th>21</th>
<th>200</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.50</td>
<td>3.50</td>
<td>5.25</td>
<td>50.00</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ears</th>
<th>13</th>
<th>14</th>
<th>21</th>
<th>208</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.12</td>
<td>3.36</td>
<td>5.04</td>
<td>49.92</td>
<td></td>
</tr>
</tbody>
</table>

- If you use \( E \) to represent the number of ears of corn and \( C \) to represent the cost for the variables instead of \( x \) and \( y \), how would you write the equations?
  - \( C = 0.25E \) and \( C = 0.24E \)

### Closing (5 minutes)

- What type of relationship can be modeled using an equation in the form \( y = kx \), and what do you need to know to write an equation in this form?
  - **A proportional relationship can be modeled using an equation in the form \( y = kx \). You need to know the constant of proportionality, which is represented by \( k \) in the equation.**
Lesson Summary

How do you find the constant of proportionality? Divide to find the unit rate, \( \frac{y}{x} = k \).

How do you write an equation for a proportional relationship? \( y = kx \), substituting the value of the constant of proportionality in place of \( k \).

What is the structure of proportional relationship equations, and how do we use them? \( x \) and \( y \) values are always left as variables, and when one of them is known, they are substituted into \( y = kx \) to find the unknown using algebra.

Exit Ticket (5 minutes)
Lesson 9: Representing Proportional Relationships with Equations

Exit Ticket

Oscar and Maria each wrote an equation that they felt represented the proportional relationship between distance in kilometers and distance in miles. One entry in the table paired 152 km with 95 miles. If \( k \) represents the number of kilometers and \( m \) represents the number of miles, who wrote the correct equation that would relate miles to kilometers? Explain why.

Oscar wrote the equation \( k = 1.6m \), and he said that the rate \( \frac{1.6}{1} \) represents kilometers per mile.

Maria wrote the equation \( k = 0.625m \) as her equation, and she said that 0.625 represents kilometers per mile.
Exit Ticket Sample Solutions

Oscar and Maria each wrote an equation that they felt represented the proportional relationship between distance in kilometers and distance in miles. One entry in the table paired 1.52 km with 95 miles. If \( k \) represents the number of kilometers and \( m \) represents the number of miles, who wrote the correct equation that would relate kilometers to miles and why?

Oscar wrote the equation \( k = 1.6m \), and he said that the rate \( \frac{1.6}{1} \) represents kilometers per mile.

Maria wrote the equation \( k = 0.625m \) as her equation, and she said that 0.625 represents kilometers per mile.

Oscar is correct. Oscar found the unit rate to be 1.6 by dividing kilometers by miles. The rate that Oscar used represents the number of kilometers per the number of miles. However, it should be noted that the variables were not well-defined. Since we do not know which values are independent or dependent, each equation should include a definition of each variable. For example, Oscar should have defined his variables so that \( k \) represented the number of kilometers and \( m \) represented the number of miles. For Maria’s equation to be correct, she should have stated that \( k \) represents number of miles and \( m \) represents number of kilometers.

Problem Set Sample Solutions

1. A person who weighs 100 pounds on Earth weighs 16.6 lb. on the moon.
   a. Which variable is the independent variable? Explain why.
      
      Weight on Earth is the independent variable because most people do not fly to the moon to weigh themselves first. The weight on the moon depends on a person’s weight on Earth.

   b. What is an equation that relates weight on Earth to weight on the moon?
      
      \[ M = \left( \frac{16.6}{100} \right) E \]
      
      \[ M = 0.166E \]

   c. How much would a 185 pound astronaut weigh on the moon? Use an equation to explain how you know.
      
      30.71 lb.

   d. How much would a man that weighs 50 pounds on the moon weigh on Earth?
      
      301 lb.

2. Use this table to answer the following questions.

<table>
<thead>
<tr>
<th>Number of Gallons of Gas</th>
<th>Number of Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>62</td>
</tr>
<tr>
<td>4</td>
<td>124</td>
</tr>
<tr>
<td>10</td>
<td>310</td>
</tr>
</tbody>
</table>

   a. Which variable is the dependent variable and why?
      
      The number of miles driven is the dependent variable because the number of miles you can drive depends on the number of gallons of gas you have in your tank.
b. Is the number of miles driven proportionally related to the number of gallons of gas? If so, what is the equation that relates the number of miles driven to the number of gallons of gas?

Yes, the number of miles driven is proportionally related to the number of gallons of gas because every measure of gallons of gas can be multiplied by \( \frac{31}{77} \) to get every corresponding measure of miles driven. 

\[ M = \frac{31}{77} G \]

c. In any ratio relating the number of gallons of gas and the number of miles driven, will one of the values always be larger? If so, which one?

Yes, the number of miles will be larger except for the point (0, 0). The point (0, 0) means 0 miles driven uses 0 gallons of gas.

d. If the number of gallons of gas is known, can you find the number of miles driven? Explain how this value would be calculated.

Yes, multiply the constant of proportionality (31 mpg) by the number of gallons of gas.

e. If the number of miles driven is known, can you find the number of gallons of gas used? Explain how this value would be calculated.

Yes, divide the number of miles driven by the constant of proportionality (31 mpg).

f. How many miles could be driven with 18 gallons of gas?

558 miles

g. How many gallons are used when the car has been driven 18 miles?

\[
\frac{18}{31} \text{ gallons}
\]

h. How many miles have been driven when half of a gallon of gas is used?

\[
\frac{31}{2} = 15.5 \text{ miles}
\]

i. How many gallons have been used when the car has been driven for a half mile?

\[
\frac{1}{62} \text{ gallons}
\]

3. Suppose that the cost of renting a snowmobile is $37.50 for 5 hours.

a. If \( c \) represents the cost and \( h \) represents the hours, which variable is the dependent variable? Explain why.

\( c \) is the dependent variable because the cost of using the snowmobile depends on the number of hours you use it. \( c = 7.5h \)

b. What would be the cost of renting 2 snowmobiles for 5 hours?

$75
4. In Katya’s car, the number of miles driven is proportional to the number of gallons of gas used. Find the missing value in the table.

<table>
<thead>
<tr>
<th>Number of Gallons of Gas</th>
<th>Number of Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
<tr>
<td>6</td>
<td>168</td>
</tr>
<tr>
<td>8</td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>280</td>
</tr>
</tbody>
</table>

a. Write an equation that will relate the number of miles driven to the number of gallons of gas.

\[ M = 28G, \text{ where } M \text{ is the number of miles and } G \text{ is the number of gallons of gas} \]

b. What is the constant of proportionality?

28

c. How many miles could Katya go if she filled her 22-gallon tank?

61.6 miles

d. If Katya takes a trip of 600 miles, how many gallons of gas would be needed to make the trip?

21 \frac{3}{7} gallons

e. If Katya drives 224 miles during one week of commuting to school and work, how many gallons of gas would she use?

8 gallons
Lesson 9: Representing Proportional Relationships with Equations

Classwork

Example 1: Jackson’s Birdhouses

Jackson and his grandfather constructed a model for a birdhouse. Many of their neighbors offered to buy the birdhouses. Jackson decided that building birdhouses could help him earn money for his summer camp, but he is not sure how long it will take him to finish all of the requests for birdhouses. If Jackson can build 7 birdhouses in 5 hours, write an equation that will allow Jackson to calculate the time it will take him to build any given number of birdhouses, assuming he works at a constant rate.

a. Write an equation that you could use to find out how long it will take him to build any number of birdhouses.

b. How many birdhouses can Jackson build in 40 hours?

c. How long will it take Jackson to build 35 birdhouses? Use the equation from part (a) to solve the problem.

d. How long will it take to build 71 birdhouses? Use the equation from part (a) to solve the problem.
### Example 2: Al’s Produce Stand

Al’s Produce Stand sells 6 ears of corn for $1.50. Barbara’s Produce Stand sells 13 ears of corn for $3.12. Write two equations, one for each produce stand, that models the relationship between the number of ears of corn sold and the cost. Then use each equation to help complete the tables below.

| Al’s Produce Stand | | Barbara’s Produce Stand |
|--------------------|-----------------|
| **Ears** | 6 | 14 | 21 | 13 | 14 | 21 |
| **Cost** | $1.50 | | $50.00 | | $3.12 | | $49.92 |
Lesson Summary

How do you find the constant of proportionality? Divide to find the unit rate, \( \frac{y}{x} = k \).

How do you write an equation for a proportional relationship? \( y = kx \), substituting the value of the constant of proportionality in place of \( k \).

What is the structure of proportional relationship equations and how do we use them? \( x \) and \( y \) values are always left as variables and when one of them is known, they are substituted into \( y = kx \) to find the unknown using algebra.

Problem Set

1. A person who weighs 100 pounds on Earth weighs 16.6 lb. on the moon.
   a. Which variable is the independent variable? Explain why.
   b. What is an equation that relates weight on Earth to weight on the moon?
   c. How much would a 185 pound astronaut weigh on the moon? Use an equation to explain how you know.
   d. How much would a man that weighs 50 pounds on the moon weigh on Earth?

2. Use this table to answer the following questions.

<table>
<thead>
<tr>
<th>Number of Gallons of Gas</th>
<th>Number of Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>62</td>
</tr>
<tr>
<td>4</td>
<td>124</td>
</tr>
<tr>
<td>10</td>
<td>310</td>
</tr>
</tbody>
</table>

   a. Which variable is the dependent variable and why?
   b. Is the number of miles driven proportionally related to the number of gallons of gas consumed? If so, what is the equation that relates the number of miles driven to the number of gallons of gas?
   c. In any ratio relating the number of gallons of gas and the number of miles driven, will one of the values always be larger? If so, which one?
   d. If the number of gallons of gas is known, can you find the number of miles driven? Explain how this value would be calculated.
   e. If the number of miles driven is known, can you find the number of gallons of gas consumed? Explain how this value would be calculated.
   f. How many miles could be driven with 18 gallons of gas?
   g. How many gallons are used when the car has been driven 18 miles?
   h. How many miles have been driven when half of a gallon of gas is used?
   i. How many gallons of gas have been used when the car has been driven for a half mile?
3. Suppose that the cost of renting a snowmobile is $37.50 for 5 hours.
   a. If \( c \) represents the cost and \( h \) represents the hours, which variable is the dependent variable? Explain why?
   b. What would be the cost of renting 2 snowmobiles for 5 hours?

4. In Katya’s car, the number of miles driven is proportional to the number of gallons of gas used. Find the missing value in the table.

<table>
<thead>
<tr>
<th>The Number of Gallons</th>
<th>The Number of Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
<tr>
<td>6</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>280</td>
</tr>
</tbody>
</table>

   a. Write an equation that will relate the number of miles driven to the number of gallons of gas.
   b. What is the constant of proportionality?
   c. How many miles could Katya go if she filled her 22-gallon tank?
   d. If Katya takes a trip of 600 miles, how many gallons of gas would be needed to make the trip?
   e. If Katya drives 224 miles during one week of commuting to school and work, how many gallons of gas would she use?
Lesson 10: Interpreting Graphs of Proportional Relationships

Student Outcomes

- Students consolidate their understanding of equations representing proportional relationships as they interpret what points on the graph of a proportional relationship mean in terms of the situation or context of the problem, including the point \((0, 0)\).
- Students are able to identify and interpret in context the point \((1, r)\) on the graph of a proportional relationship where \(r\) is the unit rate.

Classwork

**Examples (15 minutes)**

Example 1 is a review of previously taught concepts, but the lesson will be built upon this example. Pose the challenge to the students to complete the table.

Have students work individually and then compare and critique each other’s work with a partner.

<table>
<thead>
<tr>
<th>Number of Cups of Flour</th>
<th>Number of Dozens of Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

**Example 1**

Grandma’s Special Chocolate Chip Cookie recipe, which yields 4 dozen cookies, calls for 3 cups of flour. Using this information, complete the chart:

Create a table comparing the amount of flour used to the amount of cookies.

Is the number of cookies proportional to the amount of flour used? Explain why or why not.

Yes, because there exists a constant, \(\frac{4}{3}\) or \(1 \frac{1}{3}\), such that each measure of the cups of flour multiplied by the constant gives the corresponding measure of cookies.

What is the unit rate of cookies to flour \(\frac{y}{x}\), and what is the meaning in the context of the problem?

\(1 \frac{1}{3}\), 1 \frac{1}{3} dozen cookies, or 16 cookies for 1 cup of flour

Model the relationship on a graph.

Does the graph show the two quantities being proportional to each other? Explain.

The points appear on a line that passes through the origin \((0, 0)\).

Write an equation that can be used to represent the relationship.

\[ D = \frac{1}{3} F, \quad D = 1.3F, \quad \text{or} \quad D = \frac{4}{3} F \]

\(D\) represents the number of dozens of cookies

\(F\) represents the number of cups of flour
Example 2

Below is a graph modeling the amount of sugar required to make Grandma’s Chocolate Chip Cookies.

![Graph showing sugar vs. cookies]

a. Record the coordinates from the graph in a table. What do these ordered pairs represent?

- (0, 0); 0 cups of sugar will result in 0 dozen cookies
- (2, 3); 2 cups of sugar yields 3 dozen cookies
- (4, 6); 4 cups of sugar yields 6 dozen cookies
- (8, 12); 8 cups of sugar yields 12 dozen cookies
- (12, 18); 12 cups of sugar yields 18 dozen cookies
- (16, 24); 16 cups of sugar yields 24 dozen cookies

b. Grandma has 1 remaining cup of sugar. How many cookies will she be able to make? Plot the point on the graph above.

- 1.5 dozen cookies

c. How many dozen cookies can grandma make if she has no sugar? Can you graph this on the coordinate plane provided above? What do we call this point?

- (0, 0); 0 cups of sugar will result in 0 dozen cookies. The point is called the origin.

Generate class discussion using the following questions to lead to the conclusion that the point \((1, r)\) must be on the graph and what it means.

- How is the unit rate of \(\frac{y}{x}\), or in this case \(\frac{B}{A}\), related to the graph?
  - The unit rate must be the value of the \(y\)-coordinate of the point on the graph, which has an \(x\)-coordinate of one.

- What quantity is measured along the horizontal axis?
  - The number of cups of sugar

- When you plot the ordered pair \((A, B)\), what does \(A\) represent?
  - The amount of sugar, in cups, that is needed to make \(B\) dozen cookies
Lesson 10: Interpreting Graphs of Proportional Relationships

- What quantity is measured along the vertical axis?
  - The amount of cookies (number of dozens)
- When you plot the point \((A, B)\), what does \(B\) represent?
  - The total amount of cookies, in dozens, that can be made with \(A\) cups of sugar
- What is the unit rate for this proportional relationship?
  - 1.5
- Starting at the origin, if you move one unit along the horizontal axis, how far would you have to move vertically to reach the line you graphed?
  - 1.5 units
- Continue moving one unit at a time along the horizontal axis. What distance, vertically, did you move?
  - 1.5 units
- Why are we always moving 1.5 units vertically?
  - The unit rate is 1.5 dozen cookies for every 1 cup of sugar. The vertical axis, or \(y\)-value, represents the number of dozens of cookies. Since the unit rate is 1.5, every vertical move would equal the unit rate of 1.5 units.
- Do you think the vertical move will always be equal to the rate when moving 1 unit horizontally whenever two quantities that are proportional are graphed?
  - Yes, the vertical distance will always be equal to the unit rate when moving one unit horizontally on the axis.
- Graphs of different proportional relationship have different points, but what point must be on every graph of a proportional relationship? Explain why.
  - The point \((1, r)\) or unit rate must be on every graph because the unit rate describes the change in the vertical distance for every 1 unit change in the horizontal axis.

Exercises (20 minutes)

1. The graph below shows the amount of time a person can shower with a certain amount of water.

![Graph showing the relationship between gallons of water and shower time]

   a. Can you determine by looking at the graph whether the length of the shower is proportional to the number of gallons of water? Explain how you know.

      Yes, the quantities are proportional to each other since all points lie on a line that passes through the origin \((0, 0)\).
b. How long can a person shower with 15 gallons of water? How long can a person shower with 60 gallons of water?

5 minutes, 20 minutes

c. What are the coordinates of point A? Describe point A in the context of the problem.

(30, 10). If there are 30 gallons of water, then a person can shower for 10 minutes.

d. Can you use the graph to identify the unit rate?

Since the graph is a line that passes through (0, 0) and (1, r), you can take a point on the graph, such as (15, 5) and get \( \frac{1}{3} \).

e. Plot the unit rate on the graph. Is the point on the line of this relationship?

Yes, the unit rate is a point on the graph of the relationship.

f. Write the equation to represent the relationship between the number of gallons of water used and the length of a shower.

\[ m = \frac{1}{3} g \text{, where } m \text{ represents the number of minutes and } g \text{ represents the number of gallons of water.} \]

2. Your friend uses the equation \( C = 50P \) to find the total cost, \( C \), for the number of people, \( P \), entering a local amusement park.

a. Create a table and record the cost of entering the amusement park for several different-sized groups of people.

<table>
<thead>
<tr>
<th>Number of People (( P ))</th>
<th>Total Cost (in dollars, ( C ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
</tr>
</tbody>
</table>

b. Is the cost of admission proportional to the amount of people entering the amusement park? Explain why or why not.

Yes. The cost of admission is proportional to the amount of people entering the amusement park because there exists a constant (50), such that each measure of the amount of people multiplied by the constant gives the corresponding measures of cost.

c. What is the unit rate, and what does it represent in the context of the situation?

50; 1 person costs $50

d. Sketch a graph to represent this relationship.
Lesson 10: Interpreting Graphs of Proportional Relationships

Lesson Summary

The points (0, 0) and (1, \(r\)), where \(r\) is the unit rate, will always appear on the line representing two quantities that are proportional to each other.

- The unit rate, \(r\), in the point (1, \(r\)) represents the amount of vertical increase for every horizontal increase of 1 unit on the graph.
- The point (0, 0) indicates that when there is zero amount of one quantity, there will also be zero amount of the second quantity.

These two points may not always be given as part of the set of data for a given real-world or mathematical situation, but they will always appear on the line that passes through the given data points.

Closing (5 minutes)

- What points are always on the graph of two quantities that are proportional to each other?
  - The points (0, 0) and (1, \(r\)), where \(r\) is the unit rate, are always on the graph.

- How can you use the unit rate of \(\frac{y}{x}\) to create a table, equation, or graph of a relationship of two quantities that are proportional to each other?
  - In a table, you can multiply each \(x\)-value by the unit rate to obtain the corresponding \(y\)-value, or you can divide every \(y\)-value by the unit rate to obtain the corresponding \(x\)-value. You can use the equation \(y = kx\) and replace the \(k\) with the unit rate of \(\frac{y}{x}\). In a graph, the points (1, \(r\)) and (0, 0) must appear on the line of the proportional relationship.

- How can you identify the unit rate from a table, equation, or graph?
  - From a table, you can divide each \(y\)-value by the corresponding \(x\)-value. If the ratio \(y : x\) is equivalent for the entire table, then the value of the ratio \(\frac{y}{x}\) is the unit rate, and the relationship is proportional. In an equation in the form \(y = kx\), the unit rate is the number represented by the \(k\). If a graph of a line passes through the origin and contains the point (1, \(r\)), \(r\) representing the unit rate, then the relationship is proportional.

- How do you determine the meaning of any point on a graph that represents two quantities that are proportional to each other?
  - Any point \((A, B)\) on a graph that represents a proportional relationship represents a number \(A\) corresponding to the \(x\)-axis or horizontal unit, and \(B\) corresponds to the \(y\)-axis or vertical unit.

Exit Ticket (5 minutes)

What points must be on the graph of the line if the two quantities represented are proportional to each other? Explain why and describe these points in the context of the problem.

(0, 0) and (1, 50). If 0 people enter the park, then the cost would be $0. If 1 person enters the park, the cost would be $50. For every 1-unit increase along the horizontal axis, the change in the vertical distance is 50 units.

Would the point (5, 250) be on the graph? What does this point represent in the context of the situation?

Yes, the point (5, 250) would be on the graph because 5(50) = 250. The meaning is that it would cost a total of $250 for 5 people to enter the amusement park.

A STORY OF RATIOS

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Lesson 10: Interpreting Graphs of Proportional Relationships

Exit Ticket

Great Rapids White Water Rafting Company rents rafts for $125 per hour. Explain why the point (0,0) and (1,125) are on the graph of the relationship, and what these points mean in the context of the problem.
Exit Ticket Sample Solutions

Great Rapids White Water Rafting Company rents rafts for $125 per hour. Explain why the point (0, 0) and (1, 125) are on the graph of the relationship, and what these points mean in the context of the problem.

Every graph of a proportional relationship must include the points (0, 0) and (1, r). The point (0, 0) is on the graph because 0 can be multiplied by the constant to determine the corresponding value of 0. The point (1, 125) is on the graph because it is the unit rate. On the graph, for every 1 unit change on the horizontal axis, the vertical axis will change by 125 units. The point (0, 0) means 0 hours of renting a raft would cost $0, and (1, 125) means 1 hour of renting the raft would cost $125.

Problem Set Sample Solutions

1. The graph to the right shows the relationship of the amount of time (in seconds) to the distance (in feet) run by a jaguar.
   a. What does the point (5, 290) represent in the context of the situation?
      In 5 seconds, a jaguar can run 290 feet.
   b. What does the point (3, 174) represent in the context of the situation?
      A jaguar can run 174 feet in 3 seconds.
   c. Is the distance run by the jaguar proportional to the time? Explain why or why not.
      Yes, the distance run by the jaguar is proportional to the time spent running because the graph shows a line that passes through the origin (0, 0).
   d. Write an equation to represent the distance run by the jaguar. Explain or model your reasoning.
      \[ y = 58x \]
      The constant of proportionality, or unit rate of \[ \frac{y}{x} \] is 58 and can be substituted into the equation \[ y = kx \] in place of \( k \).

2. Championship t-shirts sell for $22 each.
   a. What point(s) must be on the graph for the quantities to be proportional to each other?
      (0, 0), (1, 22)
   b. What does the ordered pair (5, 110) represent in the context of this problem?
      5 t-shirts will cost $110.
   c. How many t-shirts were sold if you spent a total of $88?
      \[ \frac{88}{22} = 4 \]
3. The graph represents the total cost of renting a car. The cost of renting a car is a fixed amount each day, regardless of how many miles the car is driven.
   a. What does the ordered pair (4, 250) represent?
      It would cost $250 to rent a car for 4 days.
   b. What would be the cost to rent the car for a week? Explain or model your reasoning.
      Since the unit rate is 62.5, the cost for a week would be $437.50.

4. Jackie is making a snack mix for a party. She is using cashews and peanuts. The table below shows the relationship of the number of packages of cashews she needs to the number of cans of peanuts she needs to make the mix.

<table>
<thead>
<tr>
<th>Packages of Cashews</th>
<th>Cans of Peanuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

   a. What points must be on the graph for the number of cans of peanuts to be proportional to the number of packages of cashews? Explain why.
      (0, 0) and (1, 2). All graphs of proportional relationships are lines that pass through the origin (0, 0) and the unit rate (1, r).
   b. Write an equation to represent this relationship.
      \( y = 2x \), where \( x \) represents the number of packages of cashews and \( y \) represents the number of cans of peanuts.
   c. Describe the ordered pair (12, 24) in the context of the problem.
      In the mixture, you will need 12 packages of cashews and 24 cans of peanuts.

5. The following table shows the amount of candy and price paid.

<table>
<thead>
<tr>
<th>Amount of Candy (in pounds)</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in dollars)</td>
<td>5</td>
<td>7.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>

   a. Is the cost of the candy proportional to the amount of candy?
      Yes, because there exists a constant, 2.5, such that each measure of the amount of candy multiplied by the constant gives the corresponding measure of cost.
   b. Write an equation to illustrate the relationship between the amount of candy and the cost.
      \( y = 2.5x \)
   c. Using the equation, predict how much it will cost for 12 pounds of candy.
      \( 2.5(12) = 30 \)
d. What is the maximum amount of candy you can buy with $60? 

\[
\frac{60}{2.5} = 24 \text{ pounds}
\]

e. Graph the relationship

![Graph showing the relationship between cost and amount of candy](image)
Lesson 10: Interpreting Graphs of Proportional Relationships

Classwork

**Example 1**

Grandma’s Special Chocolate Chip Cookie recipe, which yields 4 dozen cookies, calls for 3 cups of flour.

Using this information, complete the chart:

<table>
<thead>
<tr>
<th>Create a table comparing the amount of flour used to the amount of cookies.</th>
<th>Is the number of cookies proportional to the amount of flour used? Explain why or why not.</th>
<th>What is the unit rate of cookies to flour ($\frac{y}{x}$) and what is the meaning in the context of the problem?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model the relationship on a graph.</td>
<td>Does the graph show the two quantities being proportional to each other? Explain</td>
<td>Write an equation that can be used to represent the relationship.</td>
</tr>
</tbody>
</table>
Example 2

Below is a graph modeling the amount of sugar required to make Grandma’s Chocolate Chip Cookies.

![Graph of sugar vs. cookies](image)

a. Record the coordinates from the graph in a table. What do these ordered pairs represent?

b. Grandma has 1 remaining cup of sugar. How many dozen cookies will she be able to make? Plot the point on the graph above.

c. How many dozen cookies can grandma make if she has no sugar? Can you graph this on the coordinate plane provided above? What do we call this point?
Exercises

1. The graph below shows the amount of time a person can shower with a certain amount of water.

![Graph showing time vs. gallons of water]

a. Can you determine by looking at the graph whether the length of the shower is proportional to the number of gallons of water? Explain how you know.

b. How long can a person shower with 15 gallons of water? How long can a person shower with 60 gallons of water?

c. What are the coordinates of point \(A\)? Describe point \(A\) in the context of the problem.

d. Can you use the graph to identify the unit rate?
e. Plot the unit rate on the graph. Is the point on the line of this relationship?

f. Write the equation to represent the relationship between the number of gallons of water used and the length of a shower.

2. Your friend uses the equation \( C = 50P \) to find the total cost, \( C \), for the number of people, \( P \), entering a local amusement park.
   a. Create a table and record the cost of entering the amusement park for several different-sized groups of people.

b. Is the cost of admission proportional to the amount of people entering the amusement park? Explain why or why not.

c. What is the unit rate and what does it represent in the context of the situation?
d. Sketch a graph to represent this relationship.

![Graph](image)

e. What points must be on the graph of the line if the two quantities represented are proportional to each other? Explain why and describe these points in the context of the problem.

f. Would the point \((5,250)\) be on the graph? What does this point represent in the context of the situation?
Lesson Summary

The points (0,0) and (1, \( r \)), where \( r \) is the unit rate, will always appear on the line representing two quantities that are proportional to each other.

- The unit rate, \( r \), in the point (1, \( r \)) represents the amount of vertical increase for every horizontal increase of 1 unit on the graph.
- The point (0,0) indicates that when there is zero amount of one quantity, there will also be zero amount of the second quantity.

These two points may not always be given as part of the set of data for a given real-world or mathematical situation, but they will always appear on the line that passes through the given data points.

Problem Set

1. The graph to the right shows the relationship of the amount of time (in seconds) to the distance (in feet) run by a jaguar.
   a. What does the point (5, 290) represent in the context of the situation?
   b. What does the point (3, 174) represent in the context of the situation?
   c. Is the distance run by the jaguar proportional to the time? Explain why or why not.
   d. Write an equation to represent the distance run by the jaguar. Explain or model your reasoning.

2. Championship t-shirts sell for $22 each.
   a. What point(s) must be on the graph for the quantities to be proportional to each other?
   b. What does the ordered pair (5, 110) represent in the context of this problem?
   c. How many t-shirts were sold if you spent a total of $88?

3. The graph represents the total cost of renting a car. The cost of renting a car is a fixed amount each day, regardless of how many miles the car is driven.
   a. What does the ordered pair (4, 250) represent?
   b. What would be the cost to rent the car for a week? Explain or model your reasoning.
4. Jackie is making a snack mix for a party. She is using cashews and peanuts. The table below shows the relationship of the number of packages of cashews she needs to the number of cans of peanuts she needs to make the mix.

<table>
<thead>
<tr>
<th>Packages of Cashews</th>
<th>Cans of Peanuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

a. What points must be on the graph for the number of cans of peanuts to be proportional to the number of packages of cashews? Explain why.
b. Write an equation to represent this relationship.
c. Describe the ordered pair \((12, 24)\) in the context of the problem.

5. The following table shows the amount of candy and price paid.

<table>
<thead>
<tr>
<th>Amount of Candy (in pounds)</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ( in dollars)</td>
<td>5</td>
<td>7.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>

a. Is the cost of the candy proportional to the amount of candy?
b. Write an equation to illustrate the relationship between the amount of candy and the cost.
c. Using the equation, predict how much it will cost for 12 pounds of candy.
d. What is the maximum amount of candy you can buy with $60?
e. Graph the relationship.