Learning Task: Think like a Fruit Ninja: Cross Sections of Solids

STANDARD ADDRESSED IN THIS TASK

MCC7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

STANDARDS FOR MATHEMATICAL PRACTICE
4. Model with mathematics.
5. Use appropriate tools strategically.

COMMON MISCONCEPTIONS
- Students usually think that slicing a sphere can result in a cross-section that is the shape of an ellipse (oval). However, any way you slice a sphere, you will always get a cross section of a circle.
- It is common for students to have a hard time visualizing the difference between pyramids and prisms.

ESSENTIAL QUESTIONS:
- What two-dimensional figures can be made by slicing a cube by planes?
- What two-dimensional figures can be made by slicing: cones, prisms, cylinders, and pyramids by planes?
- What strategies will ensure that all possible cross sections of a solid have been identified?

MATERIALS:
- dough or modeling clay
- fishing line or dental floss if using modeling clay
- Optional for demonstrations: power solids, geometric shapes with nets, paper, colored water, and rice
  
  http://www.learner.org/channel/courses/learningmath/geometry/session9/part_c/index.html

GROUPING
Individual/Partner/Small Group

TASK COMMENTS
In this task, students will discover what two-dimensional figures can be made when slicing a cube by planes. There are many ways to “hook” students and increase engagement in this topic. Choose one or use a combination of the examples suggested in the introduction so that students can manipulate and interact with cross sections in different ways. You will want to choose a method of demonstration or interaction based upon the technology and resources available at
your school. Most schools have received both Power Solids and geometric shapes with nets. These will make it easy for the students to experiment with this task using colored water. If some wish to use clay or play-dough, it is suggested that they cut the solids using something like fishing line or dental floss rather than a plastic knife.

**TASK DESCRIPTION**

Prior to beginning the activity, demonstrate with students the method or methods that will be used to explore the big idea of cross sections. Choose from the list below to demonstrate the general idea of cross sections and then more specifically the many cross sections of a cube:

1. The app for called “fruit ninja” is a great fun and visual way to introduce the idea of 3D shapes (fruit) being cut by a plane (sword). What you see once the fruit is cut is the cross section (2D shape)
2. There is also a Wii game called “sword play” on Wii sports resort that involves slicing many different objects with a sword. Objects include bamboo (cylinder), toaster (rectangular prism), orange (sphere), and many others. You may want to create a list ahead of time that tells students what some of the ambiguous shapes will be called for your game’s purposes. For example, the cupcake is a bit of an odd-shape with some rounded sides. So, you can choose to have students call it a hexagonal prism or rectangular prism. There will be less arguing amongst students if you establish what these imperfect 3D shapes will be considered before playing the game. Wii consoles can be hooked up to interactive white boards or classroom televisions and students can play this game individually or they can challenge each other two at a time. It can be a big event, and can even be called a “tournament” with a bracket of players. In order to win points, teachers should require students to say the name of the 2D cross section as they slice each object. This game changes the angle at which the object is sliced. Students can earn points for speed and accuracy (recorded by the game) and saying the correct cross section for each slice (recorded by the teacher).
3. Students can be given an opportunity to explore three-dimensional shapes with their hands, in a tactile way. Each student gets a small amount of modeling clay or dough, shapes it into a cube, and then cuts the cubes with fishing line or floss in different ways to see what cross-sections can be made.
4. Plastic, transparent models of cubes that have one open-face can be filled with rice, sand, or water, and tipping the cube in different ways, the students could demonstrate the different cross sections that can be made.
5. Students could access the website: [http://www.learner.org/channel/courses/learningmath/geometry/session9/part_c/index.html](http://www.learner.org/channel/courses/learningmath/geometry/session9/part_c/index.html) and use the interactive software to illustrate some of the different possible slices.
6. Show the following video called “sections of a cube”: [http://www.youtube.com/watch?v=Rc8X1_1901Q](http://www.youtube.com/watch?v=Rc8X1_1901Q).
**TASK DESCRIPTION**

**TE: Think like a Fruit Ninja: Cross Sections of Solids**

**Part I: Cross Sections of a Cube**

1. Try to make each of the following cross sections by slicing a cube.
2. Record which of the shapes you were able to create and how you did it. If you can’t make the shape, explain why not.

**Solutions**

Solutions may vary. Here are some possible solutions that students may find.

<table>
<thead>
<tr>
<th>2-D Cross Section</th>
<th>Possible?</th>
<th>Impossible?</th>
<th>Explanation why possible or why NOT possible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Square</td>
<td>X</td>
<td></td>
<td>A square cross section can be created by slicing the cube by a plane parallel to one of its square faces.</td>
</tr>
<tr>
<td>b. Equilateral triangle</td>
<td>X</td>
<td></td>
<td>An equilateral triangle cross-section can be obtained by slicing off a corner of the cube so that the three vertices of the triangle are at the same distance from the corner.</td>
</tr>
<tr>
<td>c. Rectangle, not a square</td>
<td>X</td>
<td></td>
<td>One way to obtain a rectangle that is not a square is by slicing the cube with a plane parallel to one of its edges, but not parallel to one of its square faces.</td>
</tr>
<tr>
<td>d. Triangle, not equilateral</td>
<td>X</td>
<td></td>
<td>If we slice off a corner of a cube so that the three vertices of the triangle are not at the same distance from the corner, the resulting triangle will not be equilateral.</td>
</tr>
<tr>
<td>e. Pentagon</td>
<td>X</td>
<td></td>
<td>To get a pentagon, slice with a plane going through five of the six faces of the cube.</td>
</tr>
<tr>
<td>f. Regular hexagon</td>
<td>X</td>
<td></td>
<td>To get a regular hexagon, slice with a plane going through the center of the cube and perpendicular to an interior diagonal.</td>
</tr>
<tr>
<td>g. Hexagon, not regular</td>
<td>X</td>
<td></td>
<td>Any other slice that goes through all six square faces of the cube gives a non-regular hexagon.</td>
</tr>
<tr>
<td>h. Octagon</td>
<td>X</td>
<td></td>
<td>It is not possible to create an octagonal cross-section of a cube because a cube has only six faces.</td>
</tr>
<tr>
<td>i. Trapezoid, not a parallelogram</td>
<td>X</td>
<td></td>
<td>To create a trapezoid that is not a parallelogram, slice with a plane going through one face near a vertex through the opposite face at a different distance from the opposite vertex.</td>
</tr>
<tr>
<td>j. Parallelogram, not a rectangle</td>
<td>X</td>
<td></td>
<td>To create a non-rectangular parallelogram, slice the cube by any plane that goes through two opposite corners of the cube but not containing any other vertex of the cube.</td>
</tr>
<tr>
<td>k. circle</td>
<td>X</td>
<td></td>
<td>It is not possible to create a circular cross-section of a cube.</td>
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Part II: Cross Section of a Pyramid

In the movie, Despicable Me, an inflatable model of The Great Pyramid of Giza in Egypt is created to trick people into thinking that the actual pyramid has not been stolen. When inflated, the false Great Pyramid was 225 m high and the base was square with each side 100 m in length. Construct a model of the pyramid, with a base that is 1 inch on each side. Be sure to make the height proportional to the base just as in the real pyramid.

1. What proportion can be used in order to determine the height of your model?

\[
\frac{225 \text{ m}}{100 \text{ m}} = \frac{2.25 \text{ in.}}{1 \text{ in.}}
\]

2. What is the height of your model in inches?

2.25 in.

Suppose the pyramid is sliced by a plane parallel to the base and halfway down from the top (you can cut your model to demonstrate this slice).

3. What will be the shape of the resulting cross section? square

4. Compare the dimensions of the base of the sliced off top in comparison to the base of the original un-sliced pyramid? How many inches is each side of base of the top? Justify your answer. If you slice the pyramid halfway down from the top, you’ll have a top with a base that is half the dimension of the base of the original inflatable pyramid. The sides of the base of the top will be .5 in.

Next, the pyramid is put back together and then sliced by a plane parallel to the base and 25% of the way down from the top (you can cut your model to demonstrate this slice).

5. Compare the dimensions of the base of the new smaller sliced off top in comparison to the base of the original un-sliced pyramid? How many inches is each side of this new top? If the slice is 25% of the way down from the top, you’ll have a square base for the top with sides that are 25% of the original inflatable pyramid base. that is reduced in size from the base by 75%. Reducing 1 inch by 75% results in ¼ in. dimensions for the base of the new top

What if the slicing plane is not parallel to the base? What will the shape of the cross section be under those conditions? Trapezoid, not a parallelogram
DIFFERENTIATION

Extension

Part I

- Another version of the Wii sword play game would be to have students say the 3D shape of the object AND the 2D cross section after the slice.

- Ask students to discuss and explain whether or not it is possible to cut a cube so that the resulting cross-section is a point or a line segment. Very advanced students may arrive at the conclusion that points and lines are not two-dimensional, therefore cannot be considered a two-dimensional cross section. They can describe that slicing a cube so that only one edge would be removed would create a cross section that looks like a line, but in reality, it would be a very thin rectangle. Cutting a corner of a cube would produce a square or trapezoid so small that it will look like a point. Students can debate whether or not there is a point at which these shapes become a line segment or point and is no longer 2D. Research on exact definitions of zero-dimensional, one-dimensional, two-dimensional, three-dimensional, intersection, planes, points, line segments, and lines would help students gather evidence to defend their ideas.

- Students should become aware of cross sections throughout the world in everyday situations such as nature, food, architecture, art, etc. Perhaps keeping a log of these could be helpful throughout the unit. If technology is available, students can also participate in a photo scavenger hunt where they try to take the most pictures of examples of cross-sections found in the classroom and around the school.

Part II

The Great Pyramid of Giza in Egypt is often called one of the Seven Ancient Wonders of the world. The monument was built by the Egyptian pharaoh Khufu of the Fourth Dynasty around the year 2560 BC to serve as a tomb when he died. When it was built, the Great pyramid was 145.75 m high. The base is square with each side 231 m in length.

Construct a model of the pyramid, with a base that is 6 inches on each side. Be sure to make the height proportional to the base just as in the real pyramid.

1. Suppose the pyramid is sliced by a plane parallel to the base and halfway down from the top. What will be the shape of the top? What will the dimensions of the slice be? Justify your answer.

2. What if the slice is 15% of the way down from the top?

3. What if the slicing plane is not parallel to the base? What will the shape of the slice be under those conditions?

Solutions

The model of the pyramid should have a base that is 1 ft by 1 ft and .63 feet or about 7.5 inches high.
If you slice the pyramid halfway down from the top, you’ll have a cross-section that is square and half the dimension of the base of the pyramid. If the slice is 15% of the way down from the top, you’ll have a square that is reduced in size from the base by 85%.

Interventions

- Have students use pre-cut Styrofoam shapes (found at craft store) as stamps with paint. Students can write the name of the 3D shape, stamp the 2D cross section and then name it as well on paper. Cubes, prisms, cylinders, and pyramids should be included.
SE: Think Like a Fruit Ninja: Cross-Sections of Solids

Part I: Cross Section of a Cube

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