*TASK: FOXES AND RABBITS (Spotlight Task)*
Adapted from Illustrative Mathematics
http://www.illustrativemathematics.org/illustrations/713

**STANDARDS**
Define, evaluate, and compare functions.
MCC8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.*

**STANDARDS FOR MATHEMATICAL PRACTICE**
3. Construct viable arguments and critique the reasoning of others
6. Attend to precision.
7. Look for and make use of structure.

**ESSENTIAL QUESTIONS**
- What is a function?
- What are the characteristics of a function?
- How do you determine if relations are functions?
- How is a function different from a relation?
- Compare and contrast functions and relations.

**MATERIALS REQUIRED**
- Student copy of the task

**TIME NEEDED**
- Part of 1 class period

**TEACHER NOTES**
There is a natural (and complicated!) predator-prey relationship between the fox and rabbit populations, since foxes thrive in the presence of rabbits, and rabbits thrive in the absence of foxes. However, this relationship, as shown in the given table of values, cannot possibly be used to present either population as a function of the other. This task emphasizes the importance of the "every input has exactly one output" clause in the definition of a function, which is violated in the table of values of the two populations. Noteworthy is that since the data is a collection of input-output pairs, no verbal description of the function is given, so part of the task is processing what the "rule form" of the proposed functions would look like.
The predator-prey example of foxes and rabbits is picked up again in F-IF Foxes and Rabbits 2 and 3 where students are asked to find trigonometric functions to model the two populations as functions of time.

This task could be used early on when functions are introduced. It illustrates examples of functions as well as relationships that are not functions. It could also be used as an assessment item.

This task is adapted from "Functions Modeling Change", Connally et al, Wiley 2007.

PART 1

Students study the picture and write down three thoughts and/or possible mathematic problems on a note card.

With a partner, share ideas about the picture. What mathematical questions/problems could be posed related to the picture?

PART 2

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of \( t \) corresponds to the beginning of the month.

<table>
<thead>
<tr>
<th>( t ), month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R ), number of rabbits</td>
<td>1000</td>
<td>750</td>
<td>567</td>
<td>500</td>
<td>567</td>
<td>750</td>
<td>1000</td>
<td>1250</td>
<td>1433</td>
<td>1500</td>
<td>1433</td>
</tr>
<tr>
<td>( F ), number of foxes</td>
<td>150</td>
<td>143</td>
<td>125</td>
<td>100</td>
<td>75</td>
<td>57</td>
<td>50</td>
<td>57</td>
<td>75</td>
<td>100</td>
<td>125</td>
</tr>
</tbody>
</table>

Looking back at your list of comments, questions and mathematics problems, what additional questions might be asked regarding the rabbits and foxes?

According to the data in the table, is \( F \) a function of \( R \)?
Is \( R \) a function of \( F \)?
Are either \( R \) or \( F \) functions of \( t \)?

Explain.

**PART 3**

Students share and compare their answers to the questions in Part 2.

Were any additional questions or mathematics problems posed in Part 1 that can be answered or discussed now that were not part of the questions in Part 2?

**Solution:** Each input has exactly one output

a. The key is understanding that a function is a rule that assigns to each input exactly one output, so we will test the relationships in question according to this criterion:

For the first part, that is, for \( F \) to be a function of \( R \), we think of \( R \) as the input variable and \( F \) as the output variable, and ask ourselves the following question: Is there a rule, satisfying the definition of a function, which inputs a given rabbit population and outputs the corresponding fox population. The answer is no: We can see from the data that when \( R=1000 \), we have one instance where \( F=150 \), and another where \( F=50 \). Since this means that a single input value corresponds to more than one output value, \( F \) is not a function of \( R \). In the language of the problem's context, this says that the fox population is not completely determined by the rabbit population; during two different months there are the same number of rabbits but different numbers of foxes.

Similarly, we can see that if we consider \( F \) as our input and \( R \) as our output, we have a case where \( F=100 \) corresponds to both \( R=500 \) and \( R=1500 \), two different outputs for the same input. So \( R \) is not a function of \( F \): There are two different months which have the same number of foxes but different numbers of rabbits.

b. Letting \( t \), months, be the input, we can clearly see that there is exactly one output \( R \) for each value of \( t \). That is, the rule which assigns to a month \( t \) the population of rabbits during that month fits our definition of a function, and so \( R \) is a function of \( t \). By the same reasoning \( F \) is also a function of \( t \). Again, in the context of the situation it makes sense that at any given point in time, there is a unique number of foxes and a unique number of rabbits in the park.
*TASK: FOXES AND RABBITS*

Name _______________________________ Date ______________

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